Do Non-linearities Enhance Stability of Bipedal Locomotion?

Fabian Bauer, Hartmut Hetzler, Anna Pagel, and Wolfgang Seemann

Karlsruhe Institute of Technology (KIT)
Institute of Engineering Mechanics
Chair for Dynamics / Mechatronics
Kaiserstrasse 10, 76131 Karlsruhe, Germany
{fabian.bauer,hartmut.hetzler,wolfgang.seemann}@kit.edu
http://www.itm.kit.edu/dynamik

Abstract. This contribution deals with the question if and how non-linearities can improve the stability of bipedal locomotion. In order to investigate this issue, three nonlinear modifications of a planar point-mass model are investigated: the classical spring-mass model, a model with nonlinear leg kinematics and a model with a sophisticated nonlinear muscle model. Already the simple spring-mass model is nonlinear due to its variable stucture: this basic model will serve as a benchmark. Augmented with leg-kinematics the passive two segmented leg model incorporates geometric non-linearities. By substituting the passive spring element by muscle-dynamics, the model is extended with physical non-linearities. The orbital stability of the periodic motion is analysed using Poincaré-maps at touch down. A parameter study is carried out in order to reveal differences in performance and stability of periodic motion of the three models. The influence of the different types of non-linearities is demonstrated and discussed in detail.

Keywords: dynamic walking, nonlinear dynamics, stability of running

1 Introduction

In the field of humanoid robots, bipedal locomotion is a big challenge especially concerning stability of motion, i.e. robustness against perturbations. Nowadays this complex problem is attacked by sophisticated control strategies, which consume much computational capacity as well as energy. On the other hand, human beings move well on two legs and thereby seem to spend neither much brainpower nor much energy. Thus, it seems to be benefical to transfer principles of human locomotion to two-legged machines. Unfortunately, to a great extent these basic principles are still unknown and under debate.

However, understanding these fundamental principles will enable to derive paradigms for design and control, which allow for stable locomotion with minimal control effort. One of the accepted design paradigms is the elastic leg [1]: this basic paradigm has mainly been investigated for linear elastic behavior – to

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

what extent non-linearities influence the stability of bipedal locomotion will be addressed in this paper. For this purpose three different running models will be investigated: the basic spring-mass model as reference, a more sophisticated modification with nonlinear leg-kinematics and finally a model incorporating a nonlinear muscle model (cf. fig. 1).

2 Model

To begin with, the general framework of the considered model is outlined; later three particular modifications of this basic model are described in detail. In all these three models the total inertia of the human body is lumped in its center of mass. Since lateral motion is not the focus of this consideration, the point mass is constrained to the sagittal plane, thus undergoing only plane motion. However, this restriction is not to severe since the presented models could readily be extended by a lateral degree of freedom and be easily stabilized in this dimension as suggested by [2]. Since the joint torques as well as the rotational motion of the body are not considered in this study, the inertia of the legs can be neglected and they are represented as force elements acting on the center of mass of the entire body.

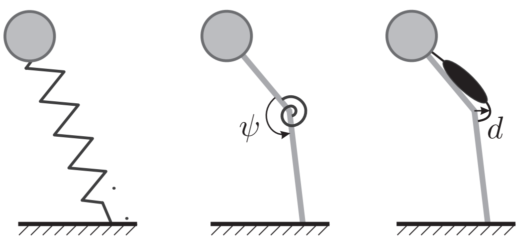


Fig. 1. Three modifications of a simple running model with different level of complexity and non-linearity: spring-mass model (*left*), passive two-segmented leg model (*middle*) and two-segmented model with muscle actuation (*right*).

During running, two different phases of motion can be distinguished, during which the degrees of freedom vary – from a mathematical point of view this implies that the equations of motion change. Therefore the system has a variable structure and thus is referred to as a hybrid dynamical system. The phases of motion are

- *Flight* as free ballistic motion in the gravitational field of the earth where the leg angle is fixed, i.e. $\varphi_0 = \text{const}$. This phase is conveniently described by the energy theorem and will not be considered in detail.
- *Stance* as bouncing and tilting over a fixed point O (see fig. 2) where the foot touches the ground. During this phase, the motion of the point of mass depends on the current leg force, which in general is a nonlinear function of the state variables (i.e. position, velocity) as well as of time.

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

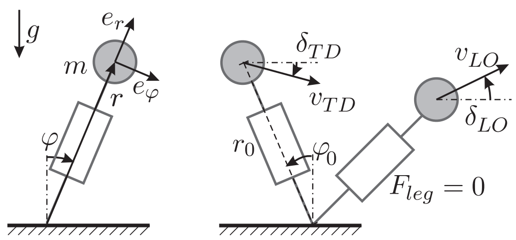


Fig. 2. Stance phase dynamics of the point mass described in polar coordinates (*left*), phase switching at predefined events (*right*).

The transition between these phases is controlled by the following conditions:

- *Flight* → *Stance:* the flight phase is terminated when the foot touches the ground ("touch down"), i.e. if during the ballistic motion the point mass reaches a certain height $h = r_0 \cos(\varphi_0)$.
- *Stance* → *Flight:* since the contact between foot and ground is unilateral (i.e. may only transmit pressure forces and no tensile forces), the transition from stance to flight occurs for vanishing leg force $F_{leg} = 0$ ("lift off").

For convenience, the equations of motion during stance are defined in polar coordinates. In dimensionless form they read

$$\rho'' = \rho\,\varphi'^2 + \kappa f_{leg}(\rho, \rho', \tau) - \gamma \cos\varphi \tag{1}$$

$$\rho\,\varphi'' = -2\rho'\varphi' + \gamma \sin\varphi\,, \tag{2}$$

where ρ denotes dimensionless leg length, φ leg angle, f_{leg} dimensionless leg force, $\tau = \frac{r_0}{v_0}$ dimensionless time and $(\,)'$ differentation with respect to dimensionless time. The dimensionless parameter $\kappa = \frac{r_0}{v_0^2 m}\hat{F}$ indicates the ratio between potential energy storable in the leg and initial touchdown kinetic energy while $\gamma = \frac{g r_0}{v_0^2}$ expresses the ratio of gravitational and kinetic energy.

2.1 Spring-Mass Model

The simplest model considered here is the spring-mass model – also known as spring loaded inverted pendulum – with a linear-elastic spring representing the effect of the leg. This simple model already predicts the global behavior (ground reaction force for instance) astonishingly well: a detailed description of the model and discussion of the results can be found in [3]. The associated dimensionless leg force is defined by the linear relation

$$f_{leg} = 1 - \rho\,. \tag{3}$$

However, the behavior of the overall model is nonlinear due to its hybrid character stemming from the switching between stance and flight phase. A possible nonlinear extension of the model is to modify the leg force according to

$$f_{leg} = (1 - \rho)^{\nu}\ . \tag{4}$$

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

Since this force law does not allow for an easy and obvious biologically motivated choice of the unknown exponent ν, it was not investigated. Instead the following model was considered, incorporating nonlinearities stemming from a more detailed modeling of human leg kinematics.

2.2 Passive Two-segmented Leg Model

The first step towards a more realistic model, capturing human physiology, is the consideration of leg-kinematics. For this purpose the human leg is represented by two massless segments of equal dimensionless length s. The segments are connected by a revolute joint and a linear-elastic torque-spring (torquefree at $\psi = \psi_0$). Since there is no actuation and only a passive force element is implemented, this model is referred to as a passive one.

Due to the nonlinear relation

$$\psi = \arccos\left(1 - \frac{1}{s^2}\rho\right) \tag{5}$$

between the compressed length ρ of the leg and the knee angle ψ the leg force for this model reads

$$f_{leg} = \frac{t}{s^2 \sin\psi(\rho)}\rho\,, \tag{6}$$

where the abbreviation $t = (\psi - \psi_0)$ for the dimensionless joint torque has been used.

In figure 3 this force law is depicted for relative leg compressions between 0 and 25 %. In the usual operating range the passive two-segmented leg shows a degressive force-compression curve. The same characteristic can be reached with the nonlinear extension of the spring-mass model described in section 2.1 by using $0 \leq \nu \leq 1$ in equation (4). A detailed description and analysis of the passive two-segmented leg can be found in [4]. So far, merely passive systems were considered in order to investigate the influence of kinematics.

2.3 Two-segmented Leg with Activated Muscle

Real world systems in general incorporate dissipation; moreover, human muscles are far from being passive elements. Hence it is obvious that a more realistic model must account for the action of muscles, including the dynamical behavior of the muscle. In order to keep the model simple, merely one virtual muscle-tendon-complex with the effect of the thigh and shank muscular system is considered. The pathway of the muscle is not modeled in detail but the force f_{mtc} produced by it acts via an effective leverarm d as joint torque t given by

$$t = d f_{mtc}\,. \tag{7}$$

This approach is well established in biomechanics. The muscle-tendon-complex consists of a serial arrangement of an elastic element (index ee) and an activated

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

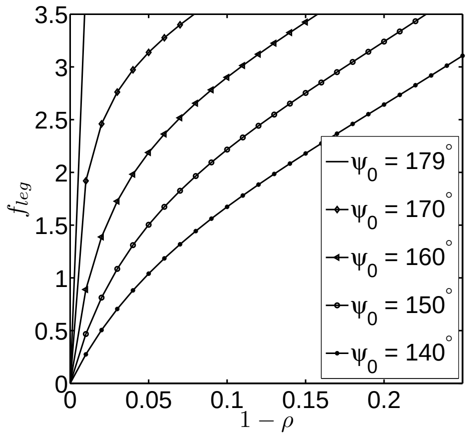


Fig. 3. Absolute value of dimensionless leg force against dimensionless leg compression of the passive two-segmented leg model with linear-elastic rotational spring.

contractile element (*ce*). The elastic element is assumed to have a quadratic force-strain relationship

$$f_{ee} = \varepsilon_{ee}^2 \tag{8}$$

and the force of the activated contractile element is assumed to depend on its actual strain ε_{ce} and strain rate $\dot{\varepsilon}_{ce}$ according to

$$f_{ce} = act \tilde{f} f(\varepsilon_{ce}) f(\dot{\varepsilon}_{ce}) \, . \tag{9}$$

Here act is the activation and $\tilde{f}$ is a dimensionless reference force. The strain ε_{ce} of the contractile element is an additional state variable. The dimensionless activation $act \in [0, 1]$ represents the state of the muscle: this is another additional state variable, and behaves like the response of an ideal lowpass to a stimulation $stim$. The stimulation is given by the control loop, which is positiv force feedback and considers signal delays τ_{delay}. Thus, the entire dynamics of stimulation and activation reads

$$T \frac{\mathrm{d}act}{\mathrm{d}\tau} + act = stim \tag{10}$$

$$stim = stim_0 + a f_{mtc} (\tau - \tau_{delay}) \, . \tag{11}$$

Finally, the resulting dimensionless muscle force f_{mtc} is computed from the equilibrium equation

$$f_{mtc} = f_{ce} = f_{ee} \, . \tag{12}$$

By muscle dynamics, three further non-linearities are added to the model: force-strain relation, force-strain rate relation and clipping of activation. In the con-

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

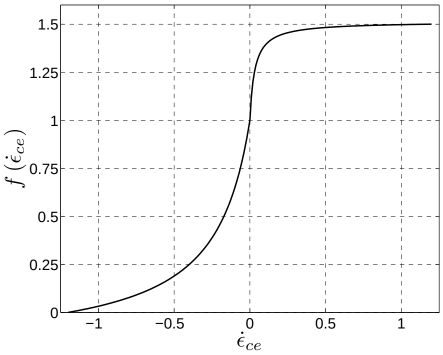


Fig. 4. Non-linear force strain-rate dependency of the contractile element. For fast contraction (*left*), force tends to zero, for fast extension, force tends assymptotically to the value of 1.5 (*right*).

sidered operating space primarily the nonlinearity of the force-strain-rate dependency (see figure 4) has an effect. The complete model is described in detail in [5], where also a biological motivation can be found.

3 Results and Discussion

In order to reveal the effects of different non-linearities, parameter studies are performed for the three proposed running models. The dimensionless notation allows for the identification of the independent parameters. Equations (1) and (2) contain the parameters κ and γ, together with the fixed leg angle during flight φ_0 there are in total three parameters to investigate. Since the three considered models have different reference forces $\hat{F}$, their κ differs. Thus, for the sake of comparability, the results of the different models are rescaled to metric quantities with units: instead of the dimensionless quantities $(\kappa, \gamma, \varphi_0)$ the three parameters (v_0, m, φ_0) are varied. The values of the remaining parameters are chosen in order to reasonably model an average adult male human being.

In order to generate results, the differential equations (1) and (2) are integrated numerically, starting from the state of touch down with a given initial speed v_0 and being continued for a defined time span. Then, Poincaré-maps of the computed data at the state of touch down are used to decide whether the motion is described by a periodical orbit. The state of touch down is defined on position level, hence only the velocity vector (i.e. value and direction) may be subject to change and has to be investigated. This vector characterises the state of the system completely.

Variations of the body mass did not show any particular impact on the behavior: therefore in figure 5 only the parameters v_0 and φ_0 are considered and shown on the horizontal axes. Figures 5(a), 5(c) and 5(e) show the velocity direction $\overline{\delta}_{TD}$ at touch down in steady state, figures 5(b), 5(d) and 5(f) the

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

corresponding velocity value $\overline{v}_{TD}$ for the respective running model. If there is no stable periodic solution – meaning that the motion sooner or later either results in stumbling or backdropping – the values are set to $\overline{\delta}_{TD} = 0$ and $\overline{v}_{TD} = 0$: in such cases stable running motion is not possible.

The total energy E_{tot} of each system consists of kinetic energy E_{kin}, gravitational potential energy E_{grav} and spring potential energy E_{spring}, i.e.

$$E_{tot}(\rho, \varphi, \rho', \varphi') = E_{kin}(\rho', \varphi') + E_{grav}(\rho, \varphi) + E_{spring}(\rho, \varphi)\ . \tag{13}$$

In the case of energy conservation (model 2.1 and 2.2) this energy is constant:

$$E_{tot}(\rho, \varphi, \rho', \varphi') = const\ . \tag{14}$$

Touch down is defined at position level (i.e. $\rho_{TD} = 1$, $\varphi_{TD} = \varphi_0$), so equation (14) yields

$$E_{kin}(\rho'_{TD}, \varphi'_{TD}) = const\ , \tag{15}$$

which means

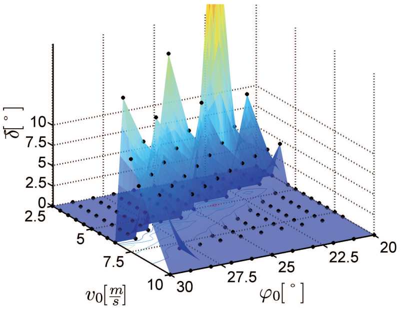
$$v_{TD} = v_0\ . \tag{16}$$

So stable running solutions of conservative systems shape an incline in the $v_0 - \varphi_0 - \overline{v}_{TD}$ space.

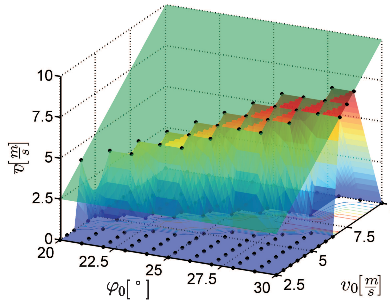
Comparing the passive two segmented model to the spring-mass model (figure 5(b)) reveals that the two-segmented leg kinematics enlarges the area of stable running solutions (figure 5(d)) at least by a factor of two. Apart from this, these two passive (i.e. conservative) models yield results that are qualitatively very similar.

In contrast to this, the activated muscle changes the characteristics of the third model significantly. The muscle is able to accelerate and decelerate the motion of the point mass. Solutions with small initial velocities no longer lead to tumbling, but are accelerated towards a periodic solution. On the other hand there are no stable solutions with velocities higher than ca. 7.5 $\frac{\text{m}}{\text{s}}$. Extended parameter studies revealed that this limit could not be moved to higher velocities by changing the parameters of the activation model. Figure 5(f) shows solutions next to periodic orbits after simulation of 100 s, so the model is able to run for quite a long time without any intelligent controller just with the simple positiv force feedback activated muscle model. As depicted in figure 4 the muscle force is increased for excentric contraction ($\dot{\varepsilon}_{ce} \geq 0$) and decreased for concentric contraction ($\dot{\varepsilon}_{ce} \leq 0$). First, as a matter of principle deceleration of the point mass in leg direction is favored compared to acceleration. In order to solve this issue the activation of the muscle has to compensate the unsymmetrical force strainrate relation by smaller values during leg shortening and bigger values during leg lengthening. The sufficient condition for stable period-1 orbits is symmetry between touch down and lift off concerning the leg angle φ ($\varphi_{TD} = -\varphi_0$, $\varphi_{LO} = \varphi_0$, $\rho(\varphi_0) = \rho(-\varphi_0)$, $\rho'(\varphi_0) = -\rho'(-\varphi_0)$, $\varphi'(\varphi_0) = \varphi'_0(-\varphi_0)$). If the muscle force and thereby the leg force can not act on the state variables to meet these conditions, there will be a continuous shift of energy towards vertical or horizontal velocity components, which leads to failure. To avoid this energy shift,

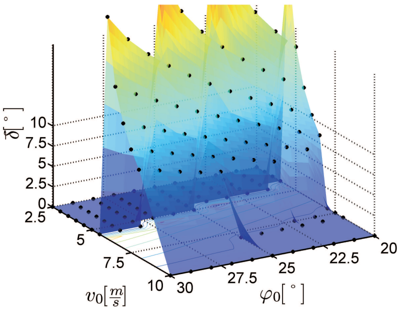
Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112



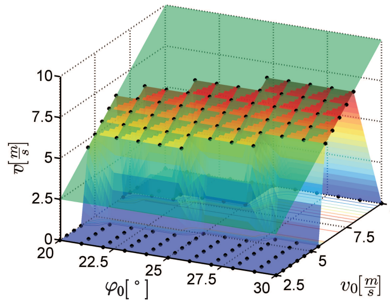
(a) Direction of the steady-state touch down velocity vector of the spring-mass model.



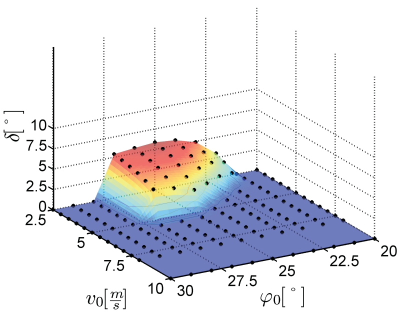
(b) Value of the steady-state touch down velocity vector of the spring-mass model.



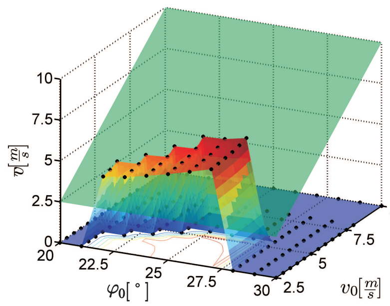
(c) Direction of the steady-state touch down velocity vector of the passive two-segmented leg model.



(d) Value of the steady-state touch down velocity vector of the passive two-segmented leg model.



(e) Direction of the steady-state touch down velocity vector of the two-segmented leg model with muscle.



(f) Value of the steady-state touch down velocity vector of the two-segmented leg model with muscle.

Fig. 5. Direction (*left*) and value (*right*) of the steady-state touch down velocity vector of the three different running models against initial velocity and touch down leg angle.

Proceedings of SIMPAR 2010 Workshops
Intl. Conf. on SIMULATION, MODELING and PROGRAMMING for AUTONOMOUS ROBOTS
Darmstadt (Germany) November 15-16, 2010
ISBN 978-3-00-032863-3
pp. 104-112

parameters of activation dynamics have to be fine tuned – for instance by an optimization tool – or adapted during action by a feedback controller.

As results have shown even the spring-mass model with a fixed leg angle during flight can asymptotically reach stable periodic solutions. The reason for the asymptotic stability of this conservative system is seen in the variable structure due to intermittent contact, which gives the piecewise holonomic system an overall non-holonomic character [7]. Furthermore it has been shown that non-linearities introduced by leg kinematics improve the stability of the basic model by enlarging the operating range of stable running. However, the break of origin symmetry in leg force – as caused by the force strain-rate relation introduced by muscle dynamics – has a rather detrimental effect on stability.

Motivated by the promising results and open questions of the active muscle model, future work will comprise more sophisticated control strategies, inspired by biology and medicine. Moreover, apart from the stability of the periodic motion the robustness of the models against single perturbations (i.e. obstacles) should be investigated. Beyond this, further research has to be done to investigate the influence of non-linearities on the stability of walking. In this context, refined models should be used since even simple experimental considerations indicate that foot and lower leg may be an important factor in understanding walking motion.

Acknowledgments. This work was funded by the German Research Foundation (DFG) within the M5 project of the Collaborative Research Center 588 'Humanoid Robots – Learning and Cooperating Multimodal Robots'.

References

1. Geyer, H., Seyfarth, A., Blickhan, R.: Compliant Leg Behaviour Explains Basic Dynamics of Walking and Running. Proc. R. Lond. B. 273, 2861–2867 (2006)
2. Seipel, J.E., Holmes, P.: Running in Three Dimensions: Analysis of a Point-mass Sprung-leg Model. Int. J. Robot. Res. 24, 657–674 (2005)
3. Blickhan, R.: The Spring-Mass Model for Running and Hopping. J. Biomech. 22, 1217–1227 (1989)
4. Rummel, J., Seyfarth, A.: Stable Running with Segmented Legs. Int. J. Robot. Res. 27, 919–934 (2008)
5. Geyer, H., Seyfarth, A., Blickhan, R.: Positive Force Feedback in Bouncing Gaits?. Proc. R. Lond. B. 270, 2173–2183 (2003)
6. Ghigliazza, R.M., Altendorfer, R., Holmes, P., Koditschek, D.: A Simply Stabilized Running Model. SIAM Review 47, 519–549 (2005)
7. Ruina, A.: Non-holonomic Stability Aspects of Piecewise Holonomic Systems. Rep. Math. Phys. 42, 343–353 (1998)