Optimization-Based Planning for Autonomous Traversal of Obstacles with Mobile Ground Robots

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ABSTRACT

Mobile robotic platforms which are traversing unstructured environments with challenging uneven terrain are permanently endangered of falling over. Previous research on trajectory planning methods for the prevention of vehicle tip-over is mostly limited to basic mobility systems with only few degrees of freedom (DOF). This paper proposes a novel optimization-based planning approach that enables mobile robots to autonomously traverse obstacles and rough terrain more safely. A 3D world model as provided from external sensors like Lidar is used to compute a whole-body motion plan in advance by optimizing the trajectories of each joint. Active flipper tracks maximize ground contact for improved traction and, if available, manipulator arm joints are used to further improve stability metrics. Additional constraints prevent collisions with the environment and the robot itself. The presented approach makes only few assumptions about the robot's configuration and is applicable to a wide range of wheeled or tracked platforms. This is demonstrated by experimental evaluation for two different robots in simulation and for one physical robot. In four different test scenarios it is shown, that the proposed approach effectively prevents vehicle tip-over during traversal of uneven ground.

Keywords: Vehicle stability, Obstacle traversal, Tip-over prevention, Flipper control

1 Introduction

In recent years, demand has been growing for mobile robotic systems that can autonomously navigate and traverse unstructured environments, e.g. a partially collapsed building after an earthquake [9]. These environments have high requirements on the mobility of the robotic platform. There is a high risk of the robot tipping over while traversing rough terrain or climbing obstacles. Falling over can damage the robot and typically leads to the end of the mission. In the worst case, the robot’s hardware may be lost.

In many situations, tipping can be prevented by reconfiguring the robot kinematics while driving. The robot can stay stable by using the arm joints to shift the center of mass (COM) and flipper joints to keep close contact with the ground. Teleoperation of such challenging maneuvers is possible. However, it requires expert skills by the operator and becomes increasingly difficult with a higher number of DOF of the robot. Therefore, only slow operation is possible under high stress for the human operator. Also the risk of erroneous movements that lead to tip-over of the robot is high.

Autonomous capabilities can reduce operator stress by controlling manipulator arm and flipper joints to automatically balance the robot based on ground geometry information and stability assessment. We propose a division of this task into two sub-problems: While pre-planning computes a trajectory for all joints based on a map of the environment, reactive behavior corrects for disturbances during execution, such as unstable ground or slippage.

This work focuses on pre-planning and presents a novel whole-body planning approach that utilizes an online-generated 3D map of the environment to optimize joint trajectories along a given path. Vehicle tip-over is prevented by assessing joint configurations with a stability metric. The
resulting motion plan allows the robot to safely traverse difficult terrain. The proposed method is applicable to a wide range of wheeled or tracked robot platforms. This is demonstrated by evaluating the performance on two different robot platforms in various scenarios in simulation and on the real robot.

The first evaluation platform Hector Tracker (Figure 1a) features one flipper track on each side of the robot, which can be adjusted in the front part and a five DOF manipulator arm. It is equipped with multiple external sensors to perceive the environment.

The second robot DRZ Telemax (Figure 1b) is a highly mobile platform with four independent flippers. The manipulator arm features six DOF. On the back, multiple sensors are mounted similar to the Hector Tracker.

Both robots are capable of generating a 3D environment map in real-time using a rotating Velodyne VLP-16 lidar.

This paper is an extension of work originally presented in the 28th International Conference on Robotics in the Alpe-Adria-Danube Region (RAAD 2019) [11]. For the first time, the extended flipper objective function is described in Section 3.3. New investigations and experimental evaluations for a second robot platform, the DRZ Telemax, are performed and reported. A detailed analysis and discussion of computational requirements in Section 4.3 gains further insights.

2 Related Work

2.1 Stability Work

Postural stability is critical for a vehicle’s ability to traverse uneven terrain. Various analytical stability margins have been proposed that predict the probability of tip-over given the vehicle location. Commonly, the calculation is based on the location of the COM and the support polygon which is defined as the convex hull of all ground contact points. A comprehensive overview and analysis of existing stability margins has been given by Garcia et al. [4]. For the analysis of quadruped walking gaits, McGhee and Frank [7] proposed a stability margin defined as the shortest distance from the COM, projected onto the plane of the support polygon, to any point on the boundary of the support polygon. An energy-based formulation was proposed by Messuri et al. [8]. Their stability margin is defined as the minimum impact energy which can be sustained by the vehicle without tipping over. Unlike the work by McGhee et al., this formulation considers COM-height changes. The force-angle stability margin was proposed by Papadopoulos et al. [15] and has been used in numerous works [2, 3, 5, 10]. It combines the distance of the COM to the support polygon edges with the angles of the gravity vector to the edges. The least stable axis determines the stability of the whole system. A positive value indicates a stable position. Similar to the energy-based formulation by Messuri et al. [8], the measure is sensitive to height changes of the COM. Furthermore, the evaluation is computationally cheap and therefore well suited for frequent evaluations within robot onboard trajectory planning. For these reasons, we utilize the force-angle stability margin in the proposed approach.

2.2 Stability Control

We categorize the existing work on methods for the prevention of vehicle tip-over on uneven terrain into two classes: Pre-planning and reactive behavior.

Reactive Behavior Grand et al. [5] optimized stability and traction of the wheel-legged robot Hylos by adjusting its posture. A different approach for the same robot was proposed by Besserer et al. [3]. They exploit redundancies of robot kinematics by decoupling control of posture and trajectory. Stability is optimized with a potential field formulation. In contrast, Ohno et al. [12] performed online reconfigurations of the flipper on a tracked vehicle to prevent tip-over around the roll axis. Ground contacts are approximated depending on flipper track contact. A different approach to flipper control has been proposed by Okada et al. [13]. They equipped their robot with two lidars that scan the ground on each side of the robot in a line. The robot posture is adjusted to match the least-squares plane estimate of the ground surface. The desired pose is evaluated with a stability margin and adjusted until a stable pose is found.

Pre-Planning Norouzi et al. [10] proposed a path planning method that also generates optimal configurations of the flipper and 1-DOF arm of the tracked iRobot Packbot. The optimization is embedded into the A* search algorithm and considers visibility, traction, energy consumption and stability. The physics engine Open Dynamics Engine (ODE) is used together with a 3D model of the environment to predict contact points. The work of Beck et al. [2] was implemented on the same platform. Given a path, the approach optimizes flipper and arm position by considering stability, equal distribution of contact forces, low energy consumption and operation within nominal joint positions.

Figure 1 Evaluation platforms Hector Tracker (a) and DRZ Telemax (b)
The prediction of contact points is idealized and depends on the flipper position and terrain slope.

The presented methods of stability control are not well suited for adoption on the Hector Tracker or DRZ Telemax. With five and six DOF respectively, the arm kinematics of each of these platforms is very complex. Therefore, further important aspects like environment- and self-collision avoidance have to be taken into account which have not been considered by previous works.

3 Whole-Body Planning

The presented approach is a generic optimization-based whole-body planner that generates motion plans to safely cross obstacles and difficult terrain for a wide range of mobile ground robots. The planner takes a path \( s \), defined by \( N_s \) equidistant waypoints, as input. Each waypoint \( s_i = (x, y, \psi)^t \), \( i = 1, ..., N_s \) is defined by a 2D-position \( (x, y) \) and a heading \( \psi \) in the xy-plane. The remaining pose components height \( z \), roll \( \phi \) and pitch \( \theta \) are constrained by the ground geometry and estimated as part of the approach. The desired path has to be generated by an external planner that is outside of the scope of this work.

3.1 Cost Function

The optimization goal is the maximization of robot stability by finding an optimal robot configuration \( \mathbf{p}_s^* \) at each pose \( s_i \) on the path. The parameter vector \( \mathbf{p} = (q_1, ..., q_{N_p}) \) specifies the flipper and manipulator arm joint angles of the robot, where \( N_p \) is the number of available DOF. It is evaluated using an appropriate objective function \( \varphi(\mathbf{p}) \), which has to be minimized:

\[
\mathbf{p}^* = \underset{\mathbf{p}}{\text{arg min}} \varphi(\mathbf{p})
\]

3.2 Optimization Constraints

To prevent damage to the robot, collisions with the environment and itself have to be avoided. This is achieved by modeling collisions as constraints of the optimization problem.

Environment Collision Avoidance The world representation used to formulate environment collision constraints is an Euclidean Signed Distance Field (ESDF) [14]. It consists of a 3-dimensional uniform grid with each cell containing the distance to the closest surface. Environmental collisions are only modeled for the manipulator arm since the flipper is intended to be in contact with the ground. Similar to [16], each link of the arm is approximated with multiple spheres (Figure 2a). One constraint per sphere is added to the optimization problem:

\[
\begin{align*}
\Phi(W_{ci,j}) - r_{i,j} > 0
\end{align*}
\]

The function \( \Phi(\cdot) \) evaluates the ESDF at the sphere center \( W_{ci,j} \) with link index \( i \) and sphere index \( j \) to get the distance to the closest obstacle. Subsequently, the radius \( r_{i,j} \) of the respective sphere is subtracted to check for a collision. Self-Collision Avoidance We distinguish between dynamic and static links to check for self-collisions efficiently. The position of dynamic links depends on the optimization parameter \( \mathbf{p} \) whereas the static part is fixed to the robot base. Analogous to the environment collision avoidance, the dynamic links of the robot are sampled with spheres (Figure 2a). One constraint per unique pair of spheres is added to the problem. The number of constraints is reduced by using the

\[
\begin{align*}
\varphi(\mathbf{p}) = \frac{1}{N_s} \sum_{i} \sum_{j} W_{ci,j} - r_{i,j} > 0
\end{align*}
\]
Allowed Collision Matrix (ACM). To prevent a collision, the distance between the sphere centers minus their radii must be greater than zero:

\[
b_{i,j,m,n}(p) = \|B_{i,j} - B_{m,n}\| - r_{i,j} - r_{m,n} > 0; i \neq m
\]  

(7)

The first link is indexed with \(i\) and its spheres with \(j\), the second link is indexed with \(m\) and its spheres with \(n\).

To reduce computational cost, static links are represented by an ESDF. It is generated by computing the distances to the links in a grid around each link (Figure 2b). Collision checks are formulated analogous to environment collision checks (Equation 6).

### 3.3 Optimization Process

In the previous Sections 3.1 and 3.2, objective function (Equation 4) and constraints (Equation 6 and 7) of the optimization have been introduced. In the following, it is presented how the function is optimized. As gradient-based optimization techniques can be faster and more efficient than gradient-free ones, a differentiable objective function is desired. In the original definition of the force-angle stability margin [15], the sign of the angle \(\theta_i\) between force vector \(\vec{f}_i\) and axis normal \(\vec{l}_i\) is given by the piecewise-defined function \(\sigma_i\). As the discontinuity is not differentiable, a signed-angle formulation is used instead:

\[
\theta_i = \text{atan2}(\vec{f}_i \times \vec{l}_i) \cdot \hat{a}_i, \ \vec{l}_i \cdot \vec{f}_i)
\]  

(8)

Computation of the stability margin relies on the contact estimation to determine robot pose and support polygon. This function space is highly complex and discontinuous. To find feasible solutions, the optimization process is separated into two consecutive phases (see Figure 3). In the first phase, only the contact geometry is optimized by adjusting the flipper position \(p_f\), eliminating the need for collision constraints. As the contact estimation is non-differentiable, only gradient-free solvers are suitable. Because we do not optimize directly for stability in this phase, an adjusted objective function \(\varphi_F(p_f)\) is used that maximizes the area \(A\) of the support polygon:

\[
\varphi_F(p_f) = -A + K_i \varphi_t(p_f) + K_c \varphi_c(p_f) + K_{p_F} \varphi_p(p_f)
\]  

(9)

Additionally, two terms for assessment of ground contact are introduced with the weights \(K_i\) and \(K_c\) and a position term \(\varphi_p(p_f)\) with the weight \(K_p\).

The first term \(\varphi_t\) rewards traction with the ground.

\[
\varphi_t(p_f) = -1 - \frac{1}{N_t} \sum_{i=1}^{N_t} \max\left(-\frac{2}{\pi} \theta_i + 1, 0\right)
\]  

(10)

The term sums up a reward for each contact point of the tracks with the ground, where 0 indicates no traction and 1 full traction. The reward is normalized by the total number of sampled points on the track \(N_t\). \(\theta_i\) is the angle between the negative gravity vector and the surface normal \(\vec{n}_i\) at contact point \(i\):

\[
\theta_i = \cos\left((0, 0, 1)^T \vec{n}_i\right)
\]  

(11)

The second term \(\varphi_c\) punishes contact points with the chassis in order to avoid such undesired collisions.
\[ \varphi_c(p_f) = \frac{C_c}{N_c} \]  

(12)

where \( C_c \) is the number of chassis contacts and \( N_c \) the total number of sampled points on the chassis.

The third term linearly penalizes the position difference between the pose estimate \( W T_E \) of the contact estimation and the homogeneous transformation \( W T_S \) to the original waypoint \( s_i \) on the path:

\[ s_{TE} = (W T_S)^{-1} W T_E \]  

(13)

\[ \varphi_p(p_f) = \max \left( 0, \| \hat{s}_{TE} \| - c_p \right) \]  

(14)

where \( \hat{s}_{TE} \in \mathbb{R}^2 \) is the xy-translatory component of the transformation \( s_{TE} \). The constant \( c_p \) specifies the allowed deviation.

In the second phase, the COM is shifted by adjusting the manipulator arm joint angles by optimizing the original objective function (Equation 4). The ground contact geometry is assumed fixed and given by the first phase. Since the contact estimation is no longer involved, efficient gradient-based optimization algorithms can be applied that also consider the collision constraints presented in Section 3.2.

If the final optimization solution \( p^* \) is unstable, the procedure stops as a safe traversal is not possible.

4 Evaluation

The proposed whole-body planning method was implemented using ROS* as middleware. The Hector Tracker robot has been used for evaluation in simulation and on the real robot. Tests on the DRZ Telemax were performed in simulation only. While the robot features four independent flippers, front and rear flippers are only moved pair-wise in these tests.

Effectiveness and versatility of the approach are assessed by crossing four different types of obstacles without tip-over with both robots. The commanded path is a straight line and defined with a resolution of 0.02 m. The optimized robot configurations \( p^*_i, i = 1, ..., N_s \) at each waypoint are connected by a joint trajectory which is executed in synchronization with vehicle movement. The environment map is represented by an ESDF. It is generated in real-time with Voxblox† and captured using a rotating VLP-16 lidar. Sensor noise is smoothed out by incrementally integrating laser scans into the map representation. The contact estimation operates directly on the ESDF and is based on iterative optimization. Derivatives of objective and constraint functions were determined with automatic differentiation using RBDL [6] and Ceres [1].

In Section 4.1 and 4.2, the predicted stability margin during planning is compared to the actual stability margin which is measured during trajectory execution. In simulation, the ground truth is used as robot pose. On the real robot, a SLAM estimate based on fusing track odometry, IMU and lidar data is used. This is followed by an analysis and discussion of the required computation time of a whole-body motion plan on the DRZ Telemax in Section 4.3.

4.1 Simulation

Experiments in simulation were performed in the open-source robotics simulator Gazebo‡. Four different testing scenarios were chosen to cover a variety of obstacles: A step with a height of 0.15 m, a double ramp with an inclination angle of 40°, a cinder block with a height of 0.14 m and an asymmetrical step with the same height as the normal step (Figure 4).

In case of the Hector Tracker, the results are compared to a scripted behavior which switches hand-tuned joint configurations based on IMU feedback. No such behavior exists for the DRZ Telemax. Comparing against a default configuration is not possible as the robot is unable to traverse the obstacles without reconfiguration. Therefore, no reference is available.

The results of the experiments with the Hector Tracker robot are summarized in Table I. The robot managed to traverse all four obstacles using the proposed whole-body planner, as can be seen by the positive minimum stability margin. The reference behavior failed to traverse the ramp

*ahttp://www.ros.org/

†https://github.com/ethz-asl/voxblox

‡http://gazebosim.org/
Table I - Evaluation results on the Hector Tracker. Plan labels the predicted stability margin during planning, Measured refers to the actual stability margin during execution. Measured values during execution of the reference behavior are given in the last column.

<table>
<thead>
<tr>
<th></th>
<th>Plan</th>
<th>Measured</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>avg</td>
<td>min</td>
<td>avg</td>
</tr>
<tr>
<td>Step</td>
<td>2.26</td>
<td>1.8</td>
<td>2.23</td>
</tr>
<tr>
<td>Ramp</td>
<td>1.98</td>
<td>0.63</td>
<td>1.75</td>
</tr>
<tr>
<td>Block</td>
<td>2.3</td>
<td>1.78</td>
<td>1.92</td>
</tr>
<tr>
<td>Asym.</td>
<td>1.68</td>
<td>0.87</td>
<td>1.57</td>
</tr>
<tr>
<td>Real robot</td>
<td>2.26</td>
<td>1.8</td>
<td>2.23</td>
</tr>
</tbody>
</table>

Figure 5 Stability graph of Hector Tracker in the ramp scenario.

and fell during descend on the downwards slope. Overall, the proposed approach significantly outperformed the reference behavior in all scenarios in terms of average stability margin. Differences between measured and predicted stability margin are explained by approximation effects in map data and contact estimation. Even though the robot crossed all obstacles, the minimum measured stability margin is close to zero in the ramp, cinder block and asymmetrical step scenarios. To explain this behavior, we take a closer look at the stability graph in the ramp scenario (Figure 5). A detailed visualization of the generated motion plan can be seen in Figure 6. The instabilities occur during the two transitions between slope and elevated floor at $x = 2$ and $x = 3.25$. The robot cannot keep contact to the ground with the flippers because only the front flippers can be adjusted. Due to this kinematic constraint, the robot is unable to traverse the obstacle without tipping in movement direction. As the robot tips into a stable state, this behavior is desired.

Likewise, the DRZ Telemax crossed all obstacles successfully. The detailed results can be seen in Table II. In all scenarios, average and minimum measured stability are close to the predicted ones. Compared to the Hector Tracker, the DRZ Telemax features independently adjustable front and rear flippers. These allow the robot to cross the obstacles without tipping motions. Figure 7 shows the motion plan on the asymmetrical step. It can be seen how the manipulator arm is shifted to the side to keep balance. The corresponding stability graph can be seen in Figure 8.

4.2 Real Robot
The evaluation on the real robot demonstrates, that the simulation results transfer to reality. Compared to simulation, the planner has to deal with inaccuracies of the robot model. The test was conducted using a single step with a height of $0.18$ m. The robot successfully managed to climb the step as can be seen in the motion plan visualization in Figure 9 and the measured stability margin in the last row of Table I. Even with model inaccuracies, the difference between average expected stability and measured stability is only $0.03$, demonstrating that the method successfully extends to the real robot. A full video of the experiment is available online8.

4.3 Computation Time
An important metric of the whole-body planner is the computation time to generate a plan. In this section, the computational requirements of the proposed approach are

Figure 6 Visualization of the whole-body motion plan with Hector Tracker in the ramp scenario.

Table II - Evaluation results on the DRZ Telemax. Plan labels the predicted stability margin during planning, Measured refers to the actual stability margin during execution.

<table>
<thead>
<tr>
<th></th>
<th>Plan</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>Step</td>
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<td>0.58</td>
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<tr>
<td>Cinder Block</td>
<td>1.4</td>
<td>0.9</td>
</tr>
<tr>
<td>Asym. Step</td>
<td>1.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

8https://youtu.be/dVLi2w4L3Lg
analyzed and discussed. Basis of the analysis is the DRZ Telemax in the four simulation scenarios. The results in each scenario can be seen in Table III. Computation was performed on the notebook CPU Intel i7-4710HQ at 2.5 GHz on a single core. Notably, the planning time is higher in the asymmetrical step scenario because the robot has to move the manipulator arm to the side to stay stable (see Figure 7) which requires more optimization steps. On average, the required planning time per meter is 54.6 s. This equals a maximum speed of 0.018 m s\(^{-1}\) if the plan is generated just in time. We compare this to the trajectory execution time in Table IV. On average, the robot moved with a speed of 0.033 m s\(^{-1}\). Therefore, to achieve real-time performance, a speed-up of the computation time of 1.78 is required.

With further tuning to the algorithm and updated computing hardware, this seems achievable.

5 Conclusion

In this paper, we presented a novel whole-body planning approach for the autonomous traversal of challenging terrain with mobile ground robots. Based on a 3D map of the environment and a given path, optimal joint trajectories for manipulator arm and flippers are computed to maximize a stability margin and traction. The method generalizes to arbitrary robot platforms using wheeled or tracked locomotion, as long as basic sensor requirements are satisfied. The versatility of the approach has been shown by successfully crossing diverse obstacles with two different robot platforms in simulation and a real robot. The proposed method consistently outperformed a hand-tuned reference in terms of average stability.
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