This is a preprint of a paper that appeared in the Proceedings of IEEE International Conference on Rehabilitation Robotics ICORR), 2015, Singapore

Analyzing and Considering Inertial Effects in Powered Lower Limb Prosthetic Design

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***Abstract*—Powered lower limb prostheses are designed to restore the biomechanical functionality of missing parts of their users' bodies. However, they do not yet meet the versatility and efficiency of the biological counterpart. A crucial open issue is how the prosthetic system and its actuator should be designed to achieve an energy efficient operation. This paper proposes a novel methodology for the design and optimization of elastically actuated lower limb prostheses. In contrast to other studies, actuator inertia is considered in this paper. Further, the approach considers the inertial parameters of the prosthesis after initial design to revise the requirements and redesign the system. The design procedure is described and presented for the example of a powered prosthetic knee. In this, considering actuator inertia enables to find optimal stiffness values for walking that are not to be found with common methods and altered optimal values for other gait types. Further, the consideration of the inertial properties of the pre-designed prosthesis in a gait simulation lead to distinctly lower requirements for peak power. For walking those are decreased by about 10% while in running a reduction of over 30% is observed. Analyzing those results, the potential of considering actuator and prosthetic inertia in design and thus the benefits due to the presented method are pointed out.**

I. Introduction

In the last decades, lower limb prostheses developed from passive mechanisms to adaptive or powered mechatronic systems that support their users either by adjusting their dynamic properties or by introducing power for locomotion [1]. Despite benefits of such modern prostheses in biomechanical function that have been shown for knee [1] and ankle joint [2], the versatility and efficiency of the biological counterpart is not yet met. Open issues in their design concern walking economy of the users [3], power required from the prosthesis [4], and enabling activities like stair climbing or running, which is only possible with few systems [5], [6].

In [7], publications on powered prosthetic knees are analyzed to identify design criteria and objectives. Criteria of major relevance are found to be joint or actuator velocities as in [8]–[10] as well as joint or actuator forces/torques, e. g., in [9]–[12]. Furthermore, decreasing required actuator power and reducing the energetic effort are approaches to find a technical solution meeting biomechanical requirements as in [9], [10]. Resulting design objectives are mimicking stiffness [8], [9], [13] and damping [13] characteristics as well as the kinematic functionality [8], [9], [11], [12] of the biological limb. Additionally, size or weight of prosthetic components is often required to comply with anthropometric dimensions of the user population as in [8], [11]. Due to the individual variations, sufficient structural strength to sustain the range of user weights is demanded, e. g., in [8], [11].

In the design process, biomechanical requirements regarding velocities, forces/torques, and powers/energies are usually determined from gait analysis and/or simulation considering able-bodied subjects, e. g., in [4], [5], [14]–[16]. Thus, the inertial influences of the prosthetic system are not considered. This is also the case in most approaches to the design and stiffness optimization of elastic actuators that are used due to their benefits in energy efficiency [4], [5], [14], [16]. A first consideration of the real inertial properties usually takes place in experiments with prototypes as in [3], [11], [12] and hence does not influence design before the prototype level. An approach to facilitate this are specific simulation models that consider the inertial properties of the prosthesis [17], [18]. Yet, they seem not to be used as tools for prosthetic design up to now, although this is recognized to be promising in [7], [19]. Beyond the inertial properties of the prosthetic system itself, the inertia of the actuator has been shown to have significant impact on power consumption and energy balance of powered devices with variable stiffness actuation [7]. However, this effect is also not covered in powered lower limb prosthetics design, e. g., in an established method for stiffness optimization [4], [14], [16].

This paper describes and discusses a method to consider the inertial influences of the whole system and the actuator in powered lower limb prosthetic design based on [7]. In Section II, the method is outlined and its theoretical basis and specific implementation are presented. Subsequently, an exemplary application to a knee prosthesis is described in Section III. Beyond considering results shown in [7], this study extends investigations to cover the influence of inertial effects in walking and running. Based on this, the influences of system and actuator inertia are analyzed and interpreted. Finally, the findings of this study are summarized and discussed in Section IV.

II. Design Approach

The design approach presented in this paper is derived from the application example given in [7]. It aims at designing powered lower limb prostheses with variable stiffness actuation considering inertial effects holistically. Therefore, such are considered in inverse dynamics simulations that are utilized for stiffness optimization and actuator dimensioning.

A. Procedure

Fig. 1 outlines the design procedure. In the first phase, the stiffness of the (serial) elastic prosthetic actuator is optimized with respect to actuator peak power or energy consumption based on non-amputee human data. The obtained value is considered in systems engineering of the prosthetic device and its actuation. Subsequently, the designed system is implemented in a simulation model of human gait by substituting the inertial properties of one (sound) leg with those of the conceptual prosthesis.

In the second phase, data of amputee gait is obtained by simulation with the model that includes the properties of the prosthesis. Kinematic data and ground reactions measured in human gait serve as the inputs to this inverse dynamics simulation that yields the required joint torques. Based on this, another iteration of stiffness optimization is performed, its result is considered in systems engineering, and finally leads to the powered prosthetic system.

B. Inverse Dynamics

Inverse dynamics simulations for stiffness optimization consider different types and velocities of gait as in [16]. To calculate the biomechanical loads on the knee joint during gait, human kinematic data acquired from participants without amputation for walking at $1.6\,\mathrm{m\,s^{-1}}$ and running at $2.6\,\mathrm{m\,s^{-1}}$ in [20] are utilized as trajectories representing gait of an average able-bodied person. As those are used as desired values in inverse dynamics simulations of non-amputee and prosthetic gait, the results of the latter ones resemble reconstructed physiological gait rather than pathological gait.

For subsequent systems engineering, the peak values of (knee) joint velocity $\hat{\dot{q}}_k$, torque $\hat{\tau}_k$, and mechanical power $\hat{P}_{m,k}$ are required. Those are calculated from measured and simulated gait data using the inverse dynamics

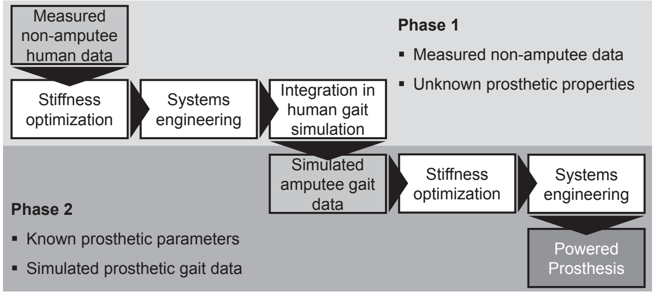

Figure 1. Design procedure based on gait measurements (phase 1) and gait simulations (phase 2).

Figure 2. Mechanical model of the considered series-elastic drive train.

models of [20] and [18]. In contrast to the elaborated model of [20], a modification of the inertial properties of the segments is possible in the one from [18].

The measured joint trajectories q_k, $\dot{q}_k$ and $\ddot{q}_k$ from [20] are substituted as link motions into the dynamics equations of the series-elastic actuator depicted in Fig. 2 (compare [7]). The stiffness of the elastic element in the drive train is denoted as K_s. Further, the knee torques obtained from inverse dynamics calculations in [20] or [18] are are considered as the elastic torque τ_k. This load comprises inertial and gravitational effects of the link as well as external reactions, such as those due to ground contact. Hence, the dynamics are modeled by

$$\tau_k + K_s\,(q_k - q_a) = 0 \tag{1}$$

$$I_a\,\ddot{q}_a + K_s\,(q_a - q_k) = \tau_a\,. \tag{2}$$

Actuator motions q_a and its derivations that are required to solve (1) and (2) are calculated using

$$q_a = q_k + K_s^{-1}\,\tau_k\,, \tag{3}$$

$$\dot{q}_a = \dot{q}_k + K_s^{-1}\,\dot{\tau}_k\,, \tag{4}$$

$$\ddot{q}_a = \ddot{q}_k + K_s^{-1}\,\ddot{\tau}_k\,. \tag{5}$$

To consider changed dynamics due to the prosthesis in phase 2 of Fig. 1, the knee torque τ_k is calculated with the model from [18] using the parameters of the prosthetic concept developed before. Additionally, the impact of considering and neglecting actuator inertia I_a is investigated by calculating required actuator power either by

$$P_{m,a} = \tau_a\,\dot{q}_a\,, \tag{6}$$

or by

$$\bar{P}_{m,a} = \tau_k\,\dot{q}_a\,, \tag{7}$$

where $\tau_k = K_s\,(q_k - q_a)$ is the elastic torque. As shown in [7], the impact of actuator inertia I_a is considered in (6) while it is neglected in (7). However, the latter one is widely used in literature, e. g., in [4], [14], [16].

Since $\dot{\tau}_k$ and $\ddot{\tau}_k$ are not contained in the data set from [20] but required in (5), those are calculated numerically from τ_k. To avoid amplification of measurement noise, the signals are lowpass filtered using a zero-lag second order Butterworth filter with a cut-off frequency of $40\,\mathrm{Hz}$ according to ones used for filtering of kinematic data in [20].

C. Optimization Method and Criteria

While optimizing for peak power or energy consumption are equally suited according to [**?**] considering the criteria individually, it is shown in [7] that appropriate compromises of peak power and energy consumption are achieved by peak power optimization. Thus, it is investigated as the main

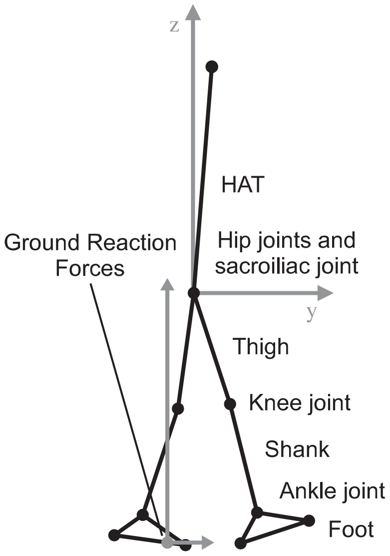

Figure 3. Biomechanical model of the human body for gait simulations.

optimization criterion aiming at an energy-efficient elastic actuator design. In heuristic optimization, serial stiffness K_s is treated as the variable parameter to determine the required stiffness bandwidth and find optimal stiffness values for certain gait types and velocities. Serial stiffness values covering the range $K_s = 100 - 1000\,\mathrm{Nm\,rad^{-1}}$ are therefore investigated in steps of $1\,\mathrm{Nm\,rad^{-1}}$.

D. Biomechanical Model and Simulation

The biomechanical model for the inverse dynamics simulation that calculates the knee torques τ_k for given joint and ground reaction force trajectories is implemented as a multi-body system within the object-oriented class library MBSLIB [18], [21]. MBSLIB applies the recursive Newton-Euler algorithm [22] to compute inverse dynamics. The biomechanical model consists of a rigid body for the trunk including arms and head and three rigid bodies for the thigh, shank and foot per leg. Each leg has three rotatory joints with a single degree of freedom for the hip, knee and ankle joints. In addition, there is a rotatory joint for the sacroiliac joint. With this configuration, the biomechanical model shown in Fig. 3 is able to reproduce fundamental human locomotion dynamics in sagittal plane. In the case of simulating prosthetic gait, the prosthesis is connected to the human model in the knee joint and substitutes foot, ankle, shank, and knee. The joint trajectories and ground reaction forces for inverse dynamics simulation are the mean of eleven female and ten male participants with a mean height of $1.73\,\mathrm{m}$ and a mean weight of $70.9\,\mathrm{kg}$ performing walking at $1.6\,\frac{\mathrm{m}}{\mathrm{s}}$ and running at $2.6\,\frac{\mathrm{m}}{\mathrm{s}}$ [20]. Body segment parameters are estimated with the software CALCMAN [23] and averaged according to the given gender ratio.

III. EXEMPLARY APPLICATION

To convey the idea of the design procedure given in Fig. 1, it is applied to the example of designing the actuation of a powered prosthetic knee device. A complete description of the design process is given in [7]. In this paper, specific focus is set on the description of the methodical consideration of inertial properties of the prosthesis in stiffness optimization and results are extended regarding running with the prosthesis.

A. Optimization with measured non-amputee gait data

According to phase 1 in Fig. 1, optimizations with data measured in non-amputee, able-bodied subjects [20] are performed regarding the conceptual knee prosthesis given in [7]. In a first assumption, this study approximates that a motor with a peak power of about $400\,\mathrm{W}$ should be sufficient to support walking and running gaits. The inertia I_a of this actuator is considered in inverse dynamics calculations based on (1) and (2). Fig. 4 shows peak powers related to subject weight versus stiffness acquired for walking at $1.6\,\mathrm{m\,s^{-1}}$.

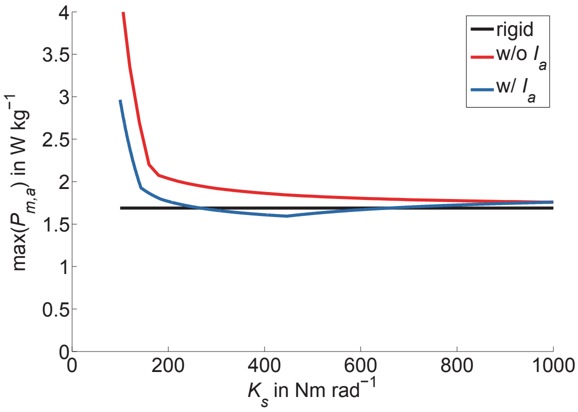

Figure 4. Peak powers related to subject weight versus stiffness during walking at $1.6\,\mathrm{m\,s^{-1}}$: Direct actuation (black) compared to SEA with I_a neglected (red) and I_a considered (blue). A power minimum is only found if I_a is considered.

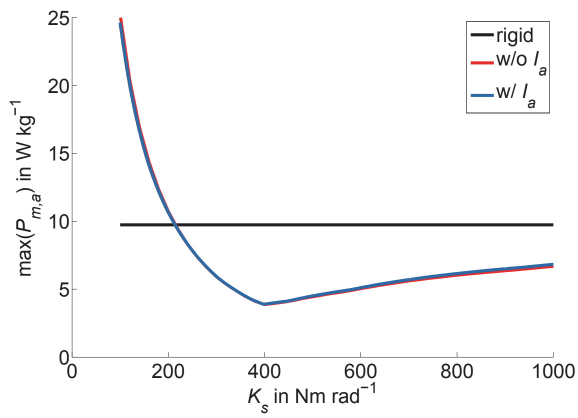

Figure 5. Peak powers related to subject weight versus stiffness during running at $2.6\,\mathrm{m\,s^{-1}}$: Direct actuation (black) compared to SEA with I_a neglected (red) and I_a considered (blue). Power minima are found irrespective of considering I_a or not.

It becomes obvious that peak power can be reduced by selecting an appropriate elasticity. If actuator intertia I_a is neglected, the optimal value is found at the maximum stiffness value since peak power converges to the one of direct drive (black). Hence, a rigid actuation concept would be recommended as in [16]. In contrast to this result seen in previous studies, peak power shows a distinct minimum when considering I_a. With a value of $1.59\,\mathrm{W\,kg^{-1}}$ at a stiffness of $446\,\mathrm{Nm\,rad^{-1}}$, it lies below the one of direct actuation at $1.69\,\mathrm{W\,kg^{-1}}$. This shows the importance of considering actuator inertia I_a in design, as a real optimum can be found for walking in contrast to the results from [16].

Results for the case of running at $2.6\,\mathrm{m\,s^{-1}}$ are shown in Fig. 5. Here, an optimal stiffness of $398\,\mathrm{Nm\,rad^{-1}}$ that shows a distinct reduction of peak power from $9.74\,\mathrm{W\,kg^{-1}}$ to $3.90\,\mathrm{W\,kg^{-1}}$ is observed. This observation is in compliance with the results from [16] and shows to occur irrespective of neglecting or considering I_a.

B. Systems Engineering

Systems engineering is performed using V model design in [7]. Based on the results of stiffness optimization, the list of requirements is updated. From that, a conceptual knee design of a powered knee device using variable torsion stiffness actuation as proposed in [24] is developed. To provide the required joint powers, a $3890048CR$ DC motor with a peak power of $406\,\mathrm{W}$ from Dr. Fritz Faulhaber GmbH & Co. KG, Schönaich, Germany is combined with a $38A$ series transmission that incorporates a reduction ratio of 60. The motor provides four-quadrant operation and drives a single-axis joint via variable torsion stiffness. The resulting design concept is depicted in Fig. 6. With its high-power actuator it enables full energy recuperation in walking and running and should support situations that exhibit high-loads (e. g., stair climbing [25]). Lithium polymer batteries are chosen as energy storage. The volume of the whole device is assessed to fit the anthropometric envelope. The weight of the knee subsystem is approximated to be $2.67\,\mathrm{kg}$. Due to stiffness optimization and energy recuperation, the concept facilitates traveling distances of more than $10\,\mathrm{km}$ by fast walking or even running. Damping properties of the knee are realized by energy recuperation through the joint actuator.

In Tab. I, the inertial and kinematic properties of the prosthesis including the foot from [26] are compared to those of the leg in the human model from [18]. In this, I_{sh} and I_{fo} denote the inertial properties of shank and foot while the corresponding masses are m_{sh} and m_{fo}. Shank length is given by l_{sh} and the center of gravity positions of the links with respect to their rotation axes and corresponding to the leg axis in standing straight are $p_{sh,a}$ and $p_{fo,a}$. The position of the center of gravity of the foot in gait direction is $p_{fo,g}$.

Using these parameters, a first approximation of the link inertia I_l occurring for the prosthesis through shank and foot with respect to the knee joint is given by

$$I_l = I_{sh} + m_{sh}\, p_{sh,a}^2 + I_{fo} + m_{fo} \left(\left(l_{sh} + p_{fo,a} \right)^2 + p_{fo,g}^2 \right). \tag{8}$$

Table I. Inertial and kinematic properties of the human leg in [18] compared to those of the conceptual prosthesis.

Parameter	Human leg	Prosthesis	Unit
I_{sh}	0.042	0.065	$\mathrm{kg\,m^2}$
I_{fo}	0.004	0.001	$\mathrm{kg\,m^2}$
m_{sh}	3.091	4.13	kg
m_{fo}	0.969	0.337	kg
l_{sh}	0.404	0.404	m
$p_{sh,a}$	0.178	0.192	m
$p_{fo,a}$	0.056	0.042	m
$p_{fo,g}$	0.060	0.027	m

C. Optimization with simulated (non-)amputee gait data

During the second phase of the design procedure outlined in Fig. 1, the real biomechanics including the proposed prosthesis are considered in a re-optimization of stiffness. Therefore, the parameters of the human leg are substituted by

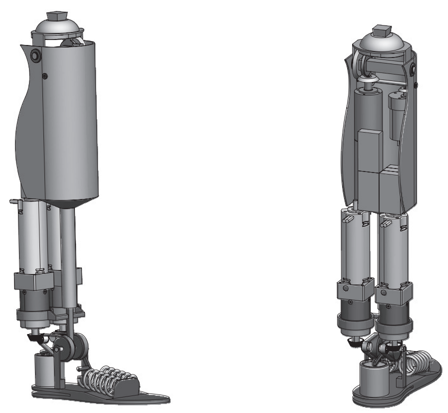

Figure 6. Front (left) and rear (right) view of a CAD model of the investigated prosthetic knee concept from [7] and the foot from [26].

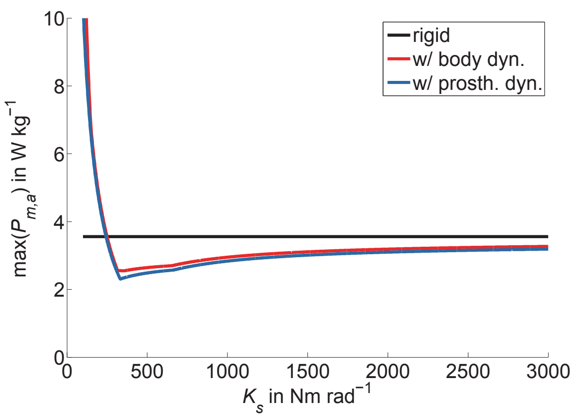

Figure 7. Peak powers related to subject weight versus stiffness during walking at $1.6\,\mathrm{m\,s^{-1}}$: Direct actuation (black) compared to SEA substituting non-amputee leg (red) and prosthetic (blue) inertial parameters in I_l. Results indicate that power consumption is reduced when considering prosthetic parameters.

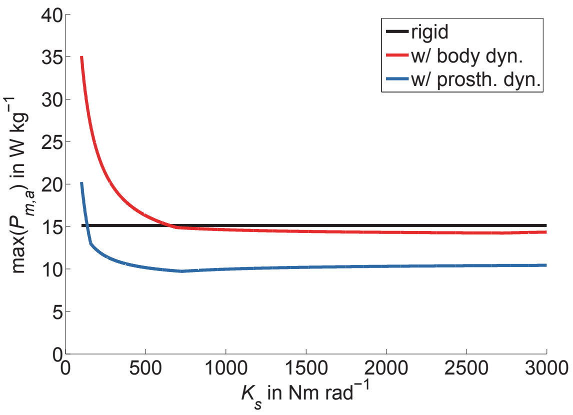

Figure 8. Peak powers related to subject weight versus stiffness during running at $2.6\,\mathrm{m\,s^{-1}}$: Direct actuation (black) compared to SEA substituting non-amputee leg (red) and prosthetic (blue) inertial parameters in I_l. Results indicate that power consumption is reduced when considering prosthetic parameters even more than in walking.

the ones of the prosthesis (compare Tab. I). Fig. 7 and Fig. 8 show the corresponding results for walking and running, respectively. Both plots rely on the data from [20] but torques and powers are calculated with the model from [18]. If the inertial properties of the prosthesis shown in Fig. 6 are substituted in the link inertia I_l of the right leg, the results represented by the blue lines are observed. For comparison, peak powers in rigid actuation are shown in black and those for series-elastic actuation substituting the inertial parameters of the unharmed leg into I_l are given in red. As in the previous investigation, both results obtained with elastic actuation exhibit peak power minima distinctly lower than requirements of rigid actuation.

When considering the inertial properties of the prosthesis in I_l instead of those of the unharmed leg, peak power decreases to $2.31\,\mathrm{Wkg^{-1}}$ $(163.5\,\mathrm{W})$ instead of $2.56\,\mathrm{Wkg^{-1}}$ $(180.6\,\mathrm{W})$ and power-optimal stiffness is shifted from $356\,\mathrm{Nm\,rad^{-1}}$ to $332\,\mathrm{Nm\,rad^{-1}}$ as shown in Fig. 7. The corresponding results for running are presented in Fig. 8. In this case a more distinct decrease of peak power from $14.24\,\mathrm{Wkg^{-1}}$ $(1009.1\,\mathrm{W})$ to $9.73\,\mathrm{Wkg^{-1}}$ $(689.3\,\mathrm{W})$ is found. Furthermore, power-optimal stiffness is found at $726\,\mathrm{Nm\,rad^{-1}}$ instead of $2728\,\mathrm{Nm\,rad^{-1}}$. As the optimal stiffness values for walking and running deviate distinctly irrespective of the inertial properties, the incorporation of a variable stiffness actuator seems to be reasonable.

Comparing the results shown in Fig. 4 and Fig. 5 reveals that the model from [18] calculates higher powers which is due to an overestimation of torques since motion trajectories are identical. Possible causes of these deviations might be different model parameters but could also be found in neglecting wobbling mass dynamics and model uncertainties according to [20]. Hence, the results shown in this paper show the potential of the design procedure proposed in this paper but do not calculate exact values for design purposes. Anyhow, the results substantiate that considering prosthetic dynamics in a second design phase is promising due to a distinct reduction of requirements: In walking, required actuator power is decreased by about 10% while a reduction of over 30% is found in running.

IV. Conclusion

This paper describes and applies a design approach to consider inertial effects in lower limb prosthetic design. Therefore, a first design of the prosthesis is developed based on requirements obtained in trials with unharmed human subjects. In a second step, the parameters of this first design are included in a human model to simulate gait including the prosthesis. With the corresponding results, requirements and design are revised. Since focus is set on powered prostheses that incorporate variable series-elastic actuation, the optimization of stiffness regarding gait is a crucial aspect. Therefore inverse dynamics simulations of the actuator are fed with the measured or simulated data.

For the exemplary application of the method to a prosthetic knee, stiffness values minimizing required peak powers are found and mainly confirm those reported in [16]. Yet, a minimum is found for walking if considering actuator inertia I_a is considered in contrast to the results of [16]. This shows the importance of considering actuator inertia I_a in design, as this yields an optimal stiffness for the elastic actuator instead of leading to the recommendation of rigid actuation.

Considering the link inertia I_l of the first prosthetic design in the second step, shows relevant reductions of power requirements. Required actuator power is decreased by about 10% in walking and by more than 30% in running when considering the values of a prosthesis instead of those of unharmed limbs. Further, those are accompanied by distinct changes of optimal stiffness values. Hence, a holistic modeling of the human-mechatronic system should leverage better designs through a more realistic prediction of the requirements. Since the model from [18] is based on non-amputee walking data, the results are only suitable for relative assessment. For more realistic simulations, it should be investigated how varied inertia affects behavior of users.

In future works the relation of nonlinear system dynamics and natural behavior will be tackled as proposed in [7]. Further, the simulation model from [18] will be improved and the method should be applied in the development of a real prosthesis.

Acknowledgment

The authors thank S. Lipfert for providing experimental human gait data. This work was funded by Forum for Interdisciplinary Research of Technische Universität Darmstadt.

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