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Towards the deployment of industrial robots as measurement instruments - An extended forward kinematic model incorporating geometric and nongeometric effects

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***Abstract*— In the area of mounting and spot-welding of body-in-white, absolutely accurate robots are installed as measuring instruments, replacing expensive coordinate and other external measuring machines. Measurement technologies based on industrial robots play an increasingly important role. Such applications require highly accurate robots. Prior to deployment of highly accurate robot, however, it needs to be ensured that the implemented robot model fits the real model. Robot calibration can offer a significant opportunity to improve the positioning accuracy and to cut production costs. Existing calibration approaches fail to capture geometric and elastic effects occurring in the robot forward kinematics. Therefore, in this work an extended forward kinematic model incorporating both geometric and elastic effects has been developed in which the positioning accuracy of a manipulator, with or without an accurate internal robot model in the robot controller, is improved.**

I. Background and Industrial Motivation

Automated manufacturing involves increasingly complex systems integration problems. In order to achieve a highly automated and precise performance of tasks, it is often necessary to redesign the entire work-cell and analyze the assembly process itself. The increasing task complexity of robots has raised particularly the interest in off-line programming. Off-line programming, however, cannot bring advantages as long as the internal robot model does not fit the real model. Besides, due to manufacturing competitiveness and economic criteria, the tendency to deploy industrial robots as measuring instruments in both automobile and general industry is increasing. Therefore, it is aimed to install accurate robots.

The discrepancies between the *internal kinematic model* of an uncalibrated robot and the real robot model are due to manufacturing tolerances and can be minimized by applying robot calibration methodologies. The non-ideal geometry of the links and joints of a manipulator causes **geometric errors** or deviations that are constant in all robot configurations. The geometric changes affect essential deviations in the position and orientation of the robot end effector. These errors are systematic and can be compensated with proper modifications of the robot model and a subsequent calibration procedure. The second important kind of error sources involves the **nongeometric** errors in joints and links and depends on the distortion of a manipulator's mechanical components, thermal effects, and loads as well as on the current joint angles. Elastic effects represent a particularly important source of error not only because elastic material deforms under stress, but also because the elasticity in the joints plays a crucial role in the achieved positioning accuracy.

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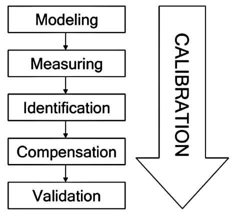

Fig. 1. Stages of the developed calibration approach.

In this paper, we propose a model-based calibration methodology consisting of five steps, none of which is trivial: modeling, measurement, identification, compensation, and validation (cf. Fig. 1).

The focus of this paper lies on the modeling step; the other steps are implicitly explained in a case study in Section V. . To the best of our knowledge, this is the first explicit development of an extended forward kinematic model that accounts for both the geometric and the nongeometric effects and that can easily be applied to any other industrial robot. Furthermore, the identifiability of both the geometric and nongeometric parameters of a highly precise model of an industrial robot arm with revolute joints is thoroughly investigated. We first survey the various kinematic modeling approaches for industrial robots with rotational joints and the usual measurement methodologies. In Section 3, we present the extended robot kinematic model. The parameter identification and the experimental setup are described in Section 4. In Section 5, numerical results attained by application of the model to a typical 6-DOF manipulator are presented.

II. Related Work

History of kinematic modeling of robots for the systematic assignment of frames to each link in a multi-link robot structure began with the famous *Denavit-Hartenberg* convention *DH*. The nominal *DH*-model is based on several simplifying assumptions such as perfect orthogonality of consecutive joint axes or the absence of manufacturing

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errors in the manipulator geometry. Improving the variations in the kinematic model arising from imprecisions in the manufacturing process is very costly. In [1] suggestions to consider the elastic deformations for serial robot models can be found. A general method to model a robot with flexible links and joints is presented in [2] . The authors argue that link deformations are not as important as joint flexibility. However, joint flexibility represented by a linear spring is only considered about the joint axis of revolute joints. In [3], the authors make use of a wrist F/T sensor and a high precision Cartesian position sensor to estimate the joint stiffness matrix. Several researchers [4], [5] consider only geometric deviations. Many approaches to robot calibration, however, do not even account for the geometric effects [6], [7]. The proposed parameter sets contain the translation parameters along *x*-, *y*-, and *z*-axis and the rotation parameters about the *x*-, *y*-, and *z*-axis of the homogeneous transformation matrix. In [4], three more parameters for the joint elasticities in *x*, *y*, *z* are considered.

In our approach we propose a fully extended model incorporating the geometric parameters with two additional distortion parameters and three parameters for the elasticities about all axes of the relevant joints. The model is easily understandable and and can be applied to any typical industrial robot with rotational joints. In total, our model extends the *DH*-convention by five additional parameters for each link. In order to ensure reliable calibration results, we set up a ranking of the various errors, also generally accepted by [8] (1) joint zero positions, (2) kinematic geometry of the robot, (3) joint elasticity, (4) other robot model parameters, (5) robot location in the work-cell, (6) robot calibration with respect to (w.r.t.) the workpiece, (7) tool calibration. In this work we consider the first three error sources, but also implicitly take into account the remaining influences. Furthermore, we avoid the use of the implemented robot error model for the calibration of the robot, as performed in [5].

The calibration of the extended model is carried out by means of an *eye-in-hand* system, i.e., a CCD camera as the only needed sensor is attached to the end effector measuring an immobile calibration object.

III. Extended Robot Kinematic Model

Based on the three basic requirements completeness, model continuity, and minimality, that every kinematic model should meet , we have developed an extended parameterized forward kinematic model for an industrial robot with revolute joints. Our initial equation to be extended is based on the *DH*-convention. To each link i, including the end effector, with i ranging from 0 to n for an n-degrees-of-freedom (DOF) manipulator, a frame S_i is attached. The final coordinate system S_n is referred to as the end effector or tool frame. The position and orientation of a reference frame S_i w.r.t. the previous reference S_{i-1} is represented by a 4×4 homogeneous matrix ${}^{i-1}T_i^{DH}$.

Each homogeneous transformation ${}^{i-1}T_i^{DH}$ is a product of

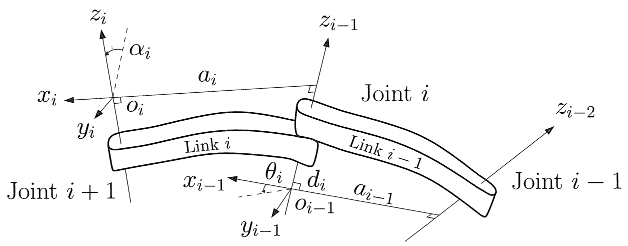

Fig. 2. Denavit-Hartenberg frame assignment [9].

four basic transformations:

$$
\begin{aligned}
{}^{i-1}T_i^{DH} &:= \mathrm{R}(z;o_i)\mathrm{Tr}(z;d_i)\mathrm{Tr}(x;a_i)\mathrm{R}(x;\alpha_i)\\
&= \begin{pmatrix} c_{o_i} & -s_{o_i}c_{\alpha_i} & s_{o_i}s_{\alpha_i} & a_i c_{o_i}\\ s_{o_i} & c_{o_i}c_{\alpha_i} & -c_{o_i}s_{\alpha_i} & a_i s_{o_i}\\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i\\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (1)
\end{aligned}
$$

with $c. = \cos(\cdot)$, $s. = \sin(\cdot)$. R represents a rotation and Tr a translation. The four quantities o_i, a_i, d_i, α_i are parameters associated with link i, $i = \{1,\ldots,n\}$ and given the names *joint offset*, *link length*, *link offset* and *link twist*. To every joint i, the corresponding joint variable q_i is associated (cf. Fig. 2). The composition of all homogeneous transformations in this kinematic chain represents the relationship of a given set of joints and the position and orientation of the end effector.

In our model, the joint offset o is modified to include the constant offset θ and the variable joint angle $\mathbf{q}$, $o_i = \theta_i + q_i$. The elements of the matrix ${}^{i-1}T_i^{DH}$ depend directly on the joint configuration parameters $\mathbf{q} \in \mathbb{R}^n$. The matrix ${}^{0}T_n^{DH}$, describing the position and orientation of the end effector frame w.r.t. the base frame, is formed by multiplying all ${}^{i-1}T_i^{DH}$ matrices in the kinematic chain from S_0 to S_n:

$$ {}^{0}T_n = {}^{0}T_1 \cdot {}^{1}T_2 \cdot \ldots \cdot {}^{n-1}T_n. $$

Note that $\cdot^{DH}$ has been omitted for brevity. In the remainder of this paper, the term *DH* is replaced by the appropriate name of the corresponding model.

A. Geometric Issues

We overcome the limitations of the *DH*-convention by defining and adding necessary parameters that account for the variations in the kinematic model. For a robot arm that has consecutive near parallel axes (z_i and z_{i+1}), the angle α_i is very small. The nominal value of this angle is zero. Consecutive rotation axes, however, are neither exactly orthogonal nor parallel (cf. Fig. 3). Consequently, two further parameters $s_{x,i}$ and $s_{y,i}$ are included in the model. These parameters describe distortions of the axis of motion z_i from its ideal orientation and are modeled as two rotations performed before the joint angle rotation in the extended kinematic model. Equation (1) is modified as follows:

$$
\begin{aligned}
{}^{i-1}T_i^{DH,s} &:= \mathrm{R}(x;s_{x,i})\mathrm{R}(y;s_{y,i})\mathrm{R}(z;q_i)\mathrm{R}(z;\theta_i)\mathrm{Tr}(z;d_i)\cdot\\
&\quad\ \mathrm{Tr}(x;a_i)\mathrm{R}(x;\alpha_i). \quad (2)
\end{aligned}
$$

B. Non-Geometric Issues

In this work, amongst the non-geometric issues the focus lies on the elasticities of the joint actuators. When applying calibration techniques, the elasticity of rotary joints, in particular those with non-vertical rotation axes, should be considered- especially when the accuracy requirements are very tight. In order to model elasticity, it is necessary to compute the static reaction forces and torques that are induced by the payload of the robot and its own body mass. It even might be necessary to produce varying reaction torques through the generation of different robot poses. Nevertheless, some robot axes such as the primary axes of SCARA robots are not identifiable. In general, the backlash or elasticity of rotary axes that are always parallel to the gravitational direction cannot be determined. Depending on the center of mass of the link, however, elasticities caused by the remaining axes might be identifiable. Therefore, we model elastic deformations as additional, configuration and parameter dependent rotations about all x-, y- and z-axes. The elastic effects in each axis depend mostly on the acting torques. The resulting rotation angles about the three axes of each link frame depend on (1) the spring constants $k_{l,i}$ for the elasticities along the joint motor drive axes, where $l \in \{x,y,z\}$, $i \in \{0,\ldots,n\}$, (2) the location of the center of mass of each link, (3) the mass m_i of the links, and (4) the current joint configuration $\mathbf{q} \in \mathbb{R}^n$ of the n actuated joint angles. Hence, for each axis of every link frame S_i, one additional rotation angle is defined: $\gamma_{l,i}$ for the rotation about the l-axis.

We assume the relationship of the elastic deformations and the moments acting on each joint axis to be linear: the joint elasticity is modeled as linear spring, similarly to the approaches in [4]. Mathematically formulated, the above angles can be determined by $\gamma_{l,i} = k_{l,i} n_{l,i}$, where the moment exerted on link i about axis l is denoted as $n_{l,i}$. The torque τ on a particle with the position $\mathbf{p}$ in any arbitrary reference frame can be defined as the cross product of $\mathbf{p}$ and the force $\mathbf{F}$ acting on the particle:

$$\tau = \mathbf{p} \times \mathbf{F}.$$

The torque τ is a vector which points along the axis of the rotation it tends to cause. It represents the magnitude of force applied to a rotational system at a distance from the axis

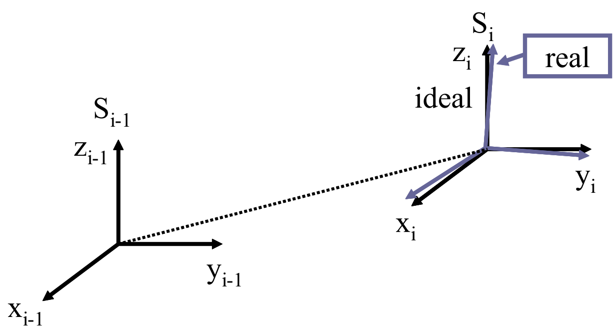

Fig. 3. Distortions in the moving axis z around the x- and y-axis.

of rotation. For the determination of the moment $n_{l,i}$, we make use of an iterative procedure in which torque $n_{l,i}$ is not directly dependent on the consecutive torque exerted on joint $n_{l,i+1}$. In general, the resulting torque on each joint axis is the sum of three partial torques, (1) the moment caused by the local load, (2) the moment caused by the load of all successive links, and (3) the free moment of the consecutive joint. We establish a compact way of formulating the calculation of the acting moments.This is particularly helpful when the location of the center of mass of the mechanical subsystems cannot be easily retrieved. In order to determine the moments, the following quantities need to be computed first:

- the mass of the mechanical subsystem composed of the manipulator links from i to j with $i < j$:

$$m_{i,j} = \sum_{k=i}^{j} m_k,$$

  where m_k is the mass of link k, and
- the center of mass of the mechanical subsystem w.r.t. the base frame:

$$\begin{aligned} {}^0\mathbf{c}_{i,j} &= \frac{1}{m_{i,j}} \sum_{k=i}^{j} m_k {}^0\mathbf{c}_k, \\ \text{where} \qquad {}^0\mathbf{c}_k &= {}^0A_k^{DH,sk}\mathbf{c}_k + {}^0\mathbf{r}_k^{DH,s}. \end{aligned}$$

${}^i\mathbf{c}_j$ denotes the center of mass of link j w.r.t. frame S_i. ${}^0A_i$ represents the rotation matrix of S_i w.r.t. S_0, and ${}^i\mathbf{r}_j$ the positioning vector of the consecutive frames S_i and S_j. Note that we use the extended forward kinematic model given in Equation (2) for the calculation of the link frames. The torque applied on the ith link within the base frame is computed by:

$$ {}^0\mathbf{N}_i = ({}^0\mathbf{c}_{i,n} - {}^0\mathbf{r}_{i-1}^{DH,s}) \times m_{i,n} {}^0\mathbf{g}_i. \tag{3}$$

${}^0\mathbf{g}_i$ represents the gravitational vector and is normalized. Finally, the acting torque in frame S_i can be computed by premultiplying Equation (3) by the necessary rotation matrices:

$$ {}^i\mathbf{N}_i = \mathrm{R}(y; s_{y,i})^T \mathrm{R}(x; s_{x,i})^{T\,i-1}A_0 {}^0\mathbf{N}_i.$$

The torque $n_{l,i}$ at each joint axis can now be easily retrieved by selecting the appropriate component:

$$n_{z,i} = (0,0,1)^i\mathbf{N}_i, \quad n_{y,i} = (0,1,0)^i\mathbf{N}_i, \quad n_{x,i} = (1,0,0)^i\mathbf{N}_i.$$

Note that we do not re-calculate the link frames based on the already determined elastic deformations since the model is already highly complex and nonlinear. Besides, in simulative tests these neglected effects proved to be ignorably small. Therefore, we consider the static, once set-up model DH,s given in Equation (2). The computation of the static torques enables the identification of the spring constants $k_{l,i}$. Concerning modeling accuracy, we like to hint to the dependencies of the static torques on the given dynamic parameters such as the mass or the center of mass of a link. These dynamic parameters usually are not exactly accurate and slightly differ from their real values.

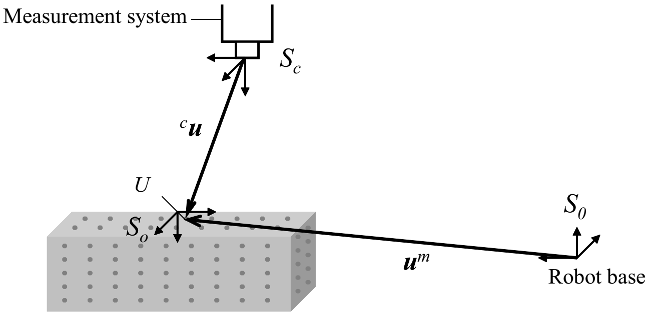

Fig. 4. An external measurement system measures the position of the point U on the calibration object. The camera attached to the robot flange measures the position of the same point w.r.t. the camera frame.

C. Complete Extended Forward Kinematic Model

Including the elastic deformations around the z_i-, y_i- and x_i-axes described by the new additional parameters $\gamma_{z,i}$, $\gamma_{y,i}$ and $\gamma_{x,i}$, the extended kinematic model *DH,s* formulated in Equation (2) can be extended further to obtain the complete extended forward kinematic model. Considering, however, the logical order of the physical effects taking place, the following logical chain of homogeneous transformations seems to be adequate:

$$\begin{aligned} {}^{i-1}T_i^{DH,s,e} \quad := \quad & \mathrm{R}(x; s_{x,i})\mathrm{R}(y; s_{y,i})\mathrm{R}(z; q_i)\mathrm{R}(z; \theta_i)\mathrm{R}(z; \gamma_{z,i}) \cdot \\ & \mathrm{R}(y; \gamma_{y,i})\mathrm{R}(x; \gamma_{x,i})\mathrm{Tr}(z; d_i)\mathrm{Tr}(x; a_i)\mathrm{R}(x; \alpha_i) \end{aligned}$$

The distortions about the x- and y- axis purpose the pre-correction of the actual link frame; thus, these two corrections are placed at the very beginning of every transformation. The rotations due to the joint elasticities are dependent on the joint configuration; hence, they are performed after the rotation about the joint angle q in each axis. For an efficient identification algorithm the extended model function's Jacobian containing the partial derivatives for all model parameters ν needs to be derived analytically. Due to the model's complexity this is no trivial process [10].

IV. Setup for Parameter Identification

A. Adaption of the Experimental Setup

We propose an experimental setup consisting of an industrial robot, a camera system that is attached to the robot flange, and appropriate calibration objects, as partially shown in Fig. 4. This setup prepares for the intended use of industrial robots as measurement tools as described in [11].

The attached camera measures the position of points on calibration objects that are placed somewhere in the workspace of the robot. The analogue information that is being calculated by the extended model *DH,s,e* is retrieved from the composition of all homogeneous transformations in the kinematic chain of the robot, the subsequent transformation from the flange frame S_n into the camera frame S_c and the transformation from the camera frame S_c into a measured point U. The calculated positioning vector of point $U$is denoted as $\mathbf{u}^{DH,s,e}(\nu,\mathbf{q}_j)$ and computed as follows:

$$\hat{\mathbf{u}}^{DH,s,e}(\nu,\mathbf{q}_j) = {}^0T_n^{DH,s,e}(\nu,\mathbf{q}_j)\,{}^nT_c\,{}^c\hat{\mathbf{u}},$$

where ν represents the parameter set and q the joint configuration. Note that $\hat{\mathbf{u}}^{DH,s,e}, {}^c\hat{\mathbf{u}} \in \mathbb{R}^4$:

$$\hat{\mathbf{u}}^{DH,s,e} = \begin{pmatrix} \mathbf{u}^{DH,s,e} \\ 1 \end{pmatrix}, \quad {}^c\hat{\mathbf{u}} = \begin{pmatrix} {}^c\mathbf{u} \\ 1 \end{pmatrix}.$$

${}^c\mathbf{u}$ represents the positioning vector of point U directly measured by the camera. The analog measured information of $\mathbf{u}^{DH,s,e}(\nu,\mathbf{q}_j)$ is represented by $\mathbf{u}_j^m$.

B. Problem Formulation

The nonlinear least squares regression function is formulated by:

$$\min_{\nu} \sum_{j=1}^{n_m} \rho_j \left\| \mathbf{u}_j^m - \mathbf{u}^{DH,s,e}(\nu,\mathbf{q}_j) \right\|_2^2,$$

where n_m stands for the number of used different joint configurations. The weights $\rho_j > 0$ may account for measurement errors if chosen different to $\rho_j = 1$.

A 1D calibration is applied when only the measured distance d^m between the robot base and the robot flange is available. The higher the number of calibrated dimensions the higher the amount of information of each measurement. Consequently, the results are more accurate and easier to retrieve.

The feasibility of multi-dimensional calibration clearly depends on both the used metrology system and the experimental setup. Obviously, a multi-dimensional calibration is more beneficial. The most important advantage is that the calibration of n dimensions requires only $\frac{1}{n}$ of the required number of measurements for the corresponding 1D calibration.

In most experimental setups, either a 6D or a 3D calibration is performed. The necessary information is usually provided by an external measurement system such as a laser tracker. Besides the orientation and position of the robot flange, further information might be available such that even more than six dimensions can be considered.

C. Initial Estimates and Boundaries

A reliable solution of the nonlinear least squares regression function requires "good" starting values for the identification parameters. According to [4] the real positioning deviations cover a quite small range of ± 1 cm at maximum. Therefore, we use as starting values for the standard *DH*-parameters the provided nominal values by the manufacturer. For all other novel additional parameters such as the distortions or elastic deformations, the starting values are set to zero as these parameters are assumed not to exist in an ideal model. Besides, we set neither lower nor upper boundaries on the angular parameters. Rather, it is necessary to set realistic boundaries on the occurring length deviations of the links since the numerical optimization method tends to compensate the positioning deviations by means of length modifications of the links [4].

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D. Sequence of Identification Steps

The parameters of the extended kinematic model cannot be calibrated at once. Instead, the influence of every single parameter regarding the positioning accuracy should be taken into account. In simulative tests, it could be observed that reliable results can be retrieved by the following sequence of identification steps:

1) The offsets θ_i of the joint zero positions are calibrated in a first step and then used as constants afterwards.
2) The remaining angular parameters $d_i, a_i, \alpha_i, s_{x,i}$, and $s_{y,i}$ are calibrated in a separate sequence. Their values are used as constants in the subsequent steps.
3) The elastic deformations $k_{x,i}, k_{y,i}$, and $k_{z,i}$ are determined in the third step.
4) Finally, all parameters are released for re-identification.

A simultaneous calibration of all parameters in a single step would neither re-identify the position of the points on the calibration objects nor the model parameters.

V. Case Study

The proposed model and parameter identification methodology was applied to the KUKA KR 125/2, an industrial robot with six revolute joints. The calibration objects, visualized in Fig. 5, consist of two sides with points marked in fixed distances of 25 mm.

The mounted CCD camera completes the kinematic chain from the robot base to the measured points on the calibration objects and can be considered as 8^{th} link connecting the robot and the calibration objects. It yields the positions of the points on the calibration objects. During the recording of one image, the robot stands still. The camera records on every side six images, i.e., each of the three blocks is captured in two different joint configurations. We assume the elastic deformations to be most important in the first three joints, particularly in the link arm and arm. We ignore any possibly elastic deformations caused by the wrist, consisting of joints 4, 5, and 6. Hence, nine nongeometric parameters are assigned to the elastic errors. The number of geometric parameters takes the value 36. For our simulative tests, the parameters of the camera frame are assumed to be quite exact. Therefore, in total 45 parameters are released for the

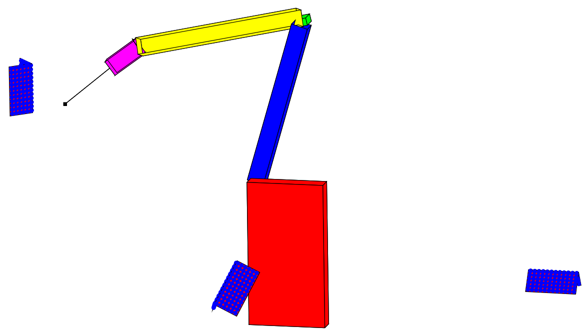

Fig. 5. Complete experimental setup.

Step	Norm of residual	Iterations
o_i	8.4207EE-7 m	10
$d_i, a_i, \alpha_i, s_{x,i}, s_{y,i}$	1.3356EE-8 m	51
$o_i, d_i, a_i, \alpha_i, s_{x,i}, s_{y,i}$	5.6802EE-16 m	61

TABLE II
COURSE OF OPTIMIZATION.

identification. For the robot specific technical data such as the *DH*- and the dynamic parameters or crucial practical questions see [10]. The used parameter deviations are based on the experiences with the approximate changes of the angular and length parameters of a typical KUKA 6-DOF industrial robot with a payload of 125 kg, gained by [4].

A. Numerical Results

The calibration procedure was validated with simulatively generated data. Our tests were conducted with different parameter sets. Also the existence of input noise on the camera measurements was considered. In Table I we present the real parameter values, the initial estimates, and the identified parameter values of one exemplary calibration of the extended model *DH,s*. The identification procedure was performed with the steps given in Table II. The plots in Fig. 6 indicate the real positioning deviations with the standard model, the positioning deviations obtained after the parameter identification of the extended model *DH,s*, and the positioning deviations obtained for the identification of the complete extended model *DH,s,e* with initial positioning deviations of about 2e-2 m.

In all tests, the end positions on the calibration objects were hit with the desired accuracy in the [1EE-9 m, 1EE-4 m] range. This accuracy range was achieved although the initial positioning deviations prior to the parameter identification was lying within the [7EE-3 m, 2EE-2 m] range. The results are also highly precise for joint configurations that are not considered in the calibration process. Note that experimental validation is of major importance and therefore still needs to be conducted.

VI. Conclusions

In this paper we established the theoretical and experimental basis for the deployment of industrial robots with revolute joints as measuring instruments taking over the role of fast and reliable warning systems. We conducted a feasibility study on achieving absolute positioning accuracy through static calibration. A parameterized extended forward kinematic model incorporating both geometric and elastic effects was developed and implemented. Furthermore, an appropriate procedure for the optimal and quick solution of the nonlinear least squares occurring in the calibration of the model parameters was developed. By means of simulatively generated data, the developed calibration procedure was applied to a typical 6-DOF industrial robot. We were able to show that the proposed model and procedure allows for the correct identification of the parameters even for joint configurations that are not considered during the calibration process.

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	Real values	Initial estimates	Identified values		Real values	Initial estimates	Identified values		Real values	Initial estimates	Identified values
o_1	0.002	0	2.1231EE-3	d_1	0.8651	0.865	0.8651	$s_{x,1}$	1EE-4	0	1.0001EE-4
o_2	0	0	-3.2701EE-8	d_2	0	0	2.5591EE-5	$s_{x,2}$	5EE-4	0	2.6182EE-4
o_3	1.572	1.5708	1.5724	d_3	0	0	2.4965EE-5	$s_{x,3}$	2EE-4	0	1.0182EE-4
o_4	0	0	-2.1644EE-4	d_4	1	1	1	$s_{x,4}$	2EE-4	0	0
o_5	0.004	0	4.4162EE-3	d_5	0	0	2.1068EE-8	$s_{x,5}$	1EE-4	0	4.8172EE-5
o_6	3.1416	3.1416	3.1416	d_6	0.20998	0.21	0.20998	$s_{x,6}$	1EE-4	0	5.1841EE-5
a_1	0.41	0.41	0.41	α_1	1.5708	1.5708	1.5711	$s_{y,1}$	0.0002	0	1.9995EE-4
a_2	1	1	1	α_2	0	0	9.998EE-5	$s_{y,2}$	3EE-4	0	1.774EE-4
a_3	4.509EE-2	0.045	4.509EE-2	α_3	1.5708	1.5708	1.5709	$s_{y,3}$	0.0001	0	9.9943EE-5
a_4	0	0	2.4181EE-8	α_4	-1.5708	-1.5708	-1.5707	$s_{y,4}$	2EE-4	0	-1.7329EE-4
a_5	0	0	8.111EE-9	α_5	1.5708	1.5708	1.5708	$s_{y,5}$	4EE-4	0	1.8346EE-4
a_6	0	0	-4.3562EE-9	α_6	0	0	3.025EE-8	$s_{y,6}$	2EE-4	0	-2.1617EE-4

TABLE I

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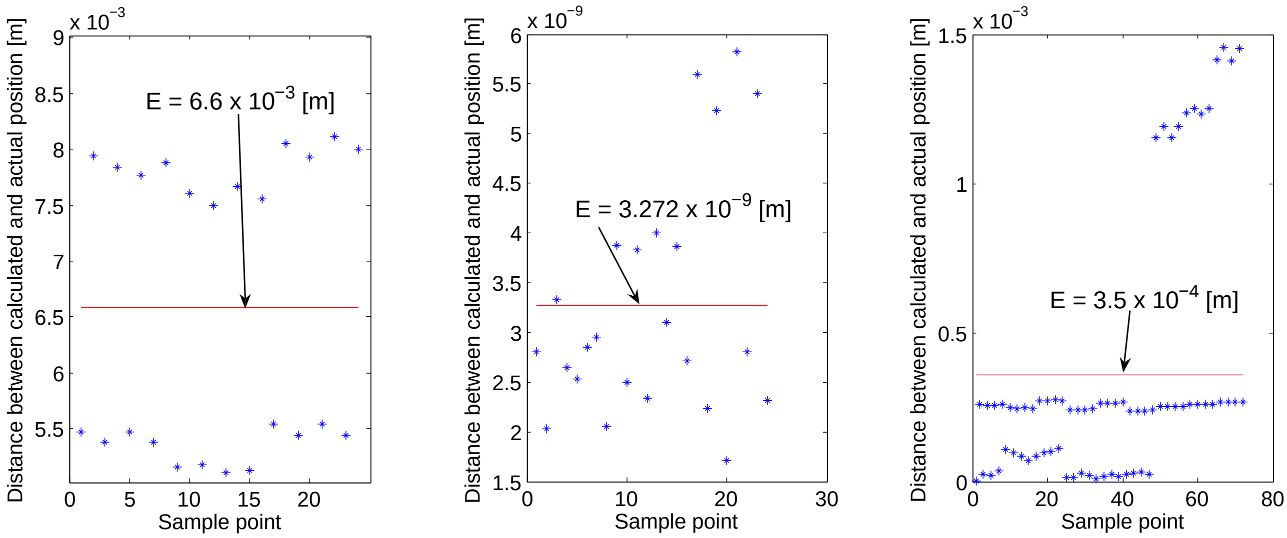

Fig. 6. From left to right: Positioning deviations obtained by the standard *DH-* model, after identification of the extended model *DH,s*, and after identification of the complete extended model *DH,s,e*.

Moreover, it could be observed that highly accurate results are achievable with the proposed model and experimental setup.

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