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Mixed-integer simulation-based optimization for a superconductive magnet design

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Abstract

The optimization of continuous parameters in electrotechnical design using electromagnetic field simulation is already standard. In this paper, we present a new sequential modelling approach for mixed-integer simulation-based optimization. We apply the method for the optimization of integer- and real-valued geometrical parameters of the coils of a superconductive magnet.

1 Introduction

The homogeneity of the magnetic field in the aperture of a superconductive magnet is determined by the geometry of the coil (Fig. 1). Especially the position of the coil blocks and the number of turns in each coil block are influencing the quality of the aperture field. The layout of the coils has to obey mechanical constraints such as, e.g., a minimal distance between two adjacent coil blocks. Invoking a separate real-valued optimization for every possible distribution of the integer number of turns over the coil blocks is not feasible. Hence, a constrained, mixed-integer nonlinear optimization has to be carried out.

In this paper a appropriate sequential modelling approach is proposed, which extends the Design and Analysis of Computer Experiments approach (DACE) of Sacks [1] handling both, the real-valued and integer variables. Also the approximation points for the model are not determined a priori, but in a sequential update process. Due to the fact, that optimization in this context is applied to a function given analytically, gradient based optimization becomes applicable.

2 Design-optimization by sequential modeling

We apply the classical way of coupling simulation and optimization for the problem considered here. The optimization supplies the design parameters for the simulation and takes the output as objective or quality function value, which is also the only provided information of the underlying physical model.

2.1 Problem formulation

For a chosen design determined by variables $s \in \Re^{n_s}$ and $p \in \mathrm{N}^{n_p}$, the quality value evaluated by magnetic field simulation has to be minimized: $\min f(s, p)$.

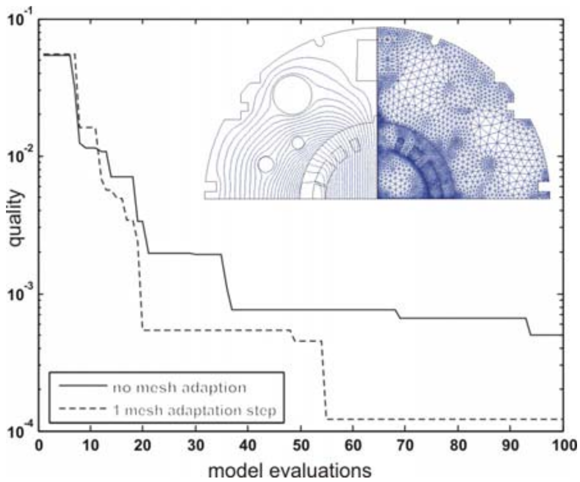

Fig. 1: Convergence of the optimization process with respect to the number of model evaluations, magnetic flux lines and adapted finite-element mesh of the magnet model.

The feasible design space for s and p, referred to as Ω, is limited by box constraints and linear constraints, $A(s, p) \le b_A$, with $b \in \Re^{n_b}$ and $A \in \Re^{n_s+n_p \times n_b}$ to meet the geometric requirements of the magnet design.

2.2 Modelling and relaxation

Given the assumption that a surrogate function $\hat{f}$ covers the main effects of f, we use an extended version of DACE for approximation, including also integer variables in an analogous way like real variables. For a given set of designs (s_i, p_i), $i = 1,...,n$, and simulation outputs $f(s_i, p_i)$, $\hat{f}$ has to meet

$$\hat{f}(s_i, p_i) = f(s_i, p_i), \quad i = 1,...,n.$$

The surrogate function is given by

$$\hat{f} = v\beta + Z(s, p),$$

where $v\beta$ covers the global trend, and $Z(s, p)$ is a realization of a stationary Gaussian function for ordinary kriging. As proposed by Sacks [1], v is equal to one and all other parameters of $Z(s, p)$, and also β

are estimated by a maximum likelihood estimation. For the known designs (s_i, p_i) the mean square error (MSE) of $\hat{f}$, which is also evaluated during the modeling process, is equal to zero. For any other design the MSE provides a measure for predicting the quality of $\hat{f}$ which is greater than zero and which is later used during the sequential update process.

This extension to include also integer variables provides a surrogate function for $\hat{f}$ that is defined not only on Ω as f but also on the continuous relaxation of $\hat{\Omega}$ regarding to s. This relaxation is not physically reasonable, but permits the use of the optimization method described below.

2.3 Optimization on metamodels

Because an evaluation of f is computationally expensive, we introduced $\hat{f}$ as a cheap and analytic surrogate function. Thanks to this and to the fact that $\hat{f}$ is also defined on $\hat{\Omega}$, the mixed-integer nonlinear programming problem can be solved by a "branch-and-bound" technique [2] which treats s to be real-valued on subproblems. These resulting nonlinear programming (NLP) subproblems of the "branch-and-bound" tree, generated by varying additional constraints to provide integrality of the solution for s, are solved by classical sequential quadratic programming techniques [3]. For instance any NLP method could also be applied on these subproblems, but in our approach gradient information of $\hat{f}$, explicitly given constraints and their gradients in case of nonlinearity, are easy to incorporate.

2.4 Sequential update process

For the initial iteration, k designs (s_l, p_l), with $l = 1,...,k$, are selected and simulated, and serve as basis B_{in} of the initial surrogate function $\hat{f}_{in}$. The minimizer of $\hat{f}_{in}$ is chosen as the next candidate (s^{*in}, p^{*in}) which is evaluated by the electromagnetic field simulation and added to B_{in} as new basis B_1 of $\hat{f}_1$. If a minimizer (s^{*j}, p^{*j}) during an iteration j is inside an $\varepsilon-$ball, $\varepsilon > 0$, around elements of B_j, the process is forced to find a design (s^{MSE*j}, p^{MSE*j}), which maximizes the MSE of $\hat{f}_j$ in order to get more information about unexplored areas of Ω. This procedure ensures that all earlier obtained information is included for the selection of new promising designs for simulation.

The update process is stopped after a given number of simulation calls depending on the provided computational resources for the optimization process.

Due to the lack of sensitivity information directly from the simulation, the parameters estimated for $Z(s, p)$ give an indication of sensitivity regarding to the different design variables. If there is a priori knowledge about sensitivity, the size of the $\varepsilon-$ball can be adjusted by different values for each dimension to catch the effects better during the update process.

3 Numerical results

The described approach is implemented using Matlab and a DACE toolbox [4], combined with an electromagnetic field simulation software [5]. For a fixed number of four coil blocks we optimize the design by continuous variation of the position vector p and the vector of numbers of turns s. One initial guess and two random chosen feasible system designs form the initial basis B_{in}. The quality improvement of the magnetic field according to the applied number of simulation calls is illustrated in Fig. 1. The numerical simulation of the system design is as a start applied for the optimization process without mesh adaptation, and a second time with one mesh adaptation step. It turns out that the best found designs of all applied optimization methods without mesh adaptation are unstable with regard to small changes in p, which indicates the necessity of mesh adaptation.

4 Conclusion

Using a sequentially updated surrogate function representing the aperture field quality of a superconductive magnet, a mixed-integer simulation-based optimization can be carried out for the coil geometry.

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6 Literature

[1] Sacks, J.; Schiller, S.B.; Welch, W.J.: Design for Computer Experiments. Technometrics Vol.31, No. 1, Feb. 1989, pp. 41 - 47

[2] Grossmann, I.E.: Review of Nonlinear Mixed-Integer and Disjunctive Programming Techniques. Optimization and Engineering. Vol. 3, 2002, pp. 227-252.

[3] Gill, P.E.; Murray, W.; Wright, M.H.: Practical Optimization. London, Academic Press. 1981.

[4] Lophaven, S.N.; Nielsen, H.B.; Søndergaard, J.: DACE, A Matlab Kriging Toolbox. Tech. Rep. IMM-TR-2002-12, Aug. 2002.

[5] Pahner, U.; Mertens, R.; De Gersem, H.; Belmans, R.; Hameyer, K.: A parametric finite element environment tuned for numerical optimization. IEEE Trans. Magn. Vol. 34, No.5, Sept. 1998, pp. 2936 - 2939.