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Development and Control of Autonomous, Biped Locomotion using Efficient Modeling, Simulation, and Optimization Techniques

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***Abstract*— Methods for modeling, simulating and optimizing the dynamics, stability and performance of legged robot locomotion are discussed in this paper. It is demonstrated how these tools are used in the design, implementation and operation of a humanoid robot. The selection and integration of fundamental hard- and software needed for autonomous operation and high agility is presented for a recently developed fully-actuated 17 DoF humanoid. The results are additionally reported from simulations and gait optimizations completed during its development using a 3D dynamic biped model coupled with multiple physical and stability constraints.**

I. Introduction

The successful development of an autonomous robot must strive for the optimal synthesis of three areas:

i) the functional and physical requirements derived from the envisaged robot application,
ii) the selection and integration of hardware (HW) and software (SW) suited to meet these requirements,
iii) the development and integration of intelligent and efficient algorithms on all levels of control, planning and perception subject to the real-time constraints given by the robot's HW/SW and its task.

Many research groups and companies are developing biped walking machines and placing effort and focus into hardware considerations. The expensive, time-consuming development and production process make it difficult though to compete with private corporations [5]. However, towards the development of an effective autonomous robot, all of the three areas mentioned above must be considered. Especially for the development of dynamic biped locomotion, we find that modeling and simulating the dynamics of biped locomotion on all levels of the design, implementation and operation phases of a humanoid robot to be important, e.g., for the selection of motors and gears, the development of joint reference trajectories for implementing first steps, the development of nonlinear dynamics model-based locomotion controllers using HW- and SW-in-the-loop environments, for task planning, and for developing autonomous behaviors.

A precise modeling of legged locomotion systems requires high dimensional nonlinear multibody systems (MBS) dynamics with constraints. Further complex tasks are generation, optimization and control of stable motions for such systems. Modeling and simulation can assist in the development of autonomous biped locomotion much more than it is currently being used. This approach complements, but not replaces the selection and integration of HW and SW.

II. Efficient Modeling and Simulation of Dynamic Biped Locomotion

A. General considerations

Biped constructions generally consist of a minimum of five bodies with two to six degrees of freedom (DoF) per leg. Dynamical simplifications allow one to analyze certain predominant behaviors of the dynamic system, but many other important features are lost. A more complete dynamical system description contains more significant dynamical effects yet a control solution for these models based on an analytical approach is usually not possible and results must be sought for numerically. The modeling and optimization approaches presented here are thus strongly dependent upon numerical methods.

Various approaches exist for modeling the MBS dynamics of a tree-structured legged robot subject to unilateral contact constraints, all with quite different characteristics regarding efficiency and accuracy in simulation and optimization. Symbolic methods are required for closed-form dynamic equations which give the best performance in terms of number of arithmetic operations and basic function evaluations needed for evaluation. This approach, though, does not fulfill the need for modularity and flexibility if parts of the kinematical structure or the kinetical data have to be changed and refined as occurs frequently during the design and operation cycle of a humanoid robot. Furthermore, it is desirable to use the same dynamic modeling framework during the entire development and operation period of a legged robot, e.g., for the selection of actuators using dynamic optimization (Sect. IV-A), and for the optimization of reference trajectories for dynamic walking (Sect. III-B), for the calibration of model parameters by optimization, for the model-based estimation of dynamic state variables, and for the future development of nonlinear dynamic model-based controllers realizing dynamically stable legged locomotion. The MBS modeling and computational approach currently used is the Articulated Body Algorithm (ABA) due to its superior modularity and computational efficiency for high dimensional systems [1], [8].

The basic equations of motion are those for a rigid, multibody system experiencing contact forces

$$\begin{aligned} \ddot{\mathbf{q}} &= \mathcal{M}(\mathbf{q})^{-1}\Big(B\mathbf{u} - \mathcal{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathcal{G}(\mathbf{q}) + J_c(\mathbf{q})^T\mathbf{f}_c\Big) \\ 0 &= \mathbf{g_c}(\mathbf{q}) \end{aligned} \quad (1)$$

where N equals the number of links in the system, $\mathcal{M} \in \mathbb{R}^{N\times N}$ is the square, positive-definite mass-inertia matrix, $\mathcal{C} \in \mathbb{R}^N$ contains the Coriolis and centrifugal forces, $\mathcal{G} \in \mathbb{R}^N$ the gravitational forces, and $\mathbf{u}(t) \in \mathbb{R}^m$ are the control input functions which are mapped with the constant matrix $B \in \mathbb{R}^{N\times m}$ to the actively controlled joints. The ground contact constraints $\mathbf{g_c} \in \mathbb{R}^{n_c}$ represent holonomic constraints on the system from which the constraint Jacobian may be obtained $J_c = \frac{\partial \mathbf{g_c}}{\partial \mathbf{q}} \in \mathbb{R}^{n_c\times N}$, while $\mathbf{f}_c \in \mathbb{R}^{n_c}$ is the ground constraint force.

A property prevalent in legged machines is that their constrained contact legs often have unique inverse kinematic solutions for their joint angles and angle velocities. This lends itself to the use of reduced dynamics algorithms for simulation and optimization. The projection of the dynamics (1) onto a reduced set of independent states converts the differential-algebraic (DAE) contact system (1) into an ODE system of minimal size. Define the independent $\mathbf{q_I}$ and dependent $\mathbf{q_D}$ states as:

$$\begin{aligned} \mathbf{q_I} &= \text{global orientation, position; swing leg(s) states} \\ \mathbf{q_D} &= \text{contact leg(s) states} \end{aligned}$$

from which $\mathbf{q_I} = Z\mathbf{q}$, where Z is a constant mapping. The reduced dynamics

$$\ddot{\mathbf{q}}_{\mathbf{I}} = Z\mathcal{M}(\mathbf{q})^{-1}\Big(B\mathbf{u} - \mathcal{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathcal{G}(\mathbf{q}) + J_c^T\mathbf{f}_c\Big) . \quad (2)$$

is computed using a recursive numerical multibody algorithm [4]. The second time derivative of the contact constraints are then satisfied with the simulation of this ODE.

An important aspect of formulating a gait optimization problem is establishing the many constraints on the problem. For a biped, the gait cycle consists of several phases describing different contact situations and being separated by events. The order of contact events is straightforward and depends primarily upon the speed of locomotion. A summary of the *physical* modeling constraints for a *half-stride* of a periodic gait cycle in 3-dimensions is [4]:

Periodic gait constraints (gait optimization):

1) Periodicity of continuous state and control variables.
2) Periodicity of ground contact forces.
3) Rotational states, controls & forces are symmetric about inertial y-axis and anti-symmetric about x- and z-axes. Linear states & forces are symmetric about inertial x- and z-axes and anti-symmetric about y-axis (See Fig. 2).

Exterior environmental constraints:

1) Ground clearance of the swing legs.
2) Ground contact forces lie within the friction cone and unilateral contact constraints are not violated and reach equality when contact is broken.
3) Each contact foot's *center of pressure* (CoP) lies in the interior of its contact surface [2].
4) Enforcement of position and velocity contact constraints at beginning and end of each phase.

Interior modeling constraints:

1) Jump conditions in the system velocities due to inelastic collisions of the legs with the ground.
2) Magnitude bounds on states, accelerations, controls.
3) Actuator torque-speed limitations.

Further constraints may be applied for speed, stability or energy consumption considerations.

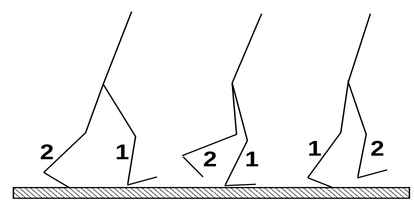

Fig. 1. Three Phases of Dynamic Gait with Different Foot Contact Positions for Leg 1: (1) Heel Roll, (2) Flat Contact, (3) Toe Roll

Depending upon whether a statically stable or dynamically stable biped gait is desired, the optimization problem formulation will have different periodicity, symmetry, and kinematic phase boundary constraints depending on the foot contact positions (Fig. 1). The number of phases may also differ. We model the static and dynamically stable gaits as follows.

Statically Stable Gait:
Phase 1: Foot 1 flat contact, Foot 2 swinging freely
Phase 2: Foot 1 flat contact, Foot 2 flat contact

Dynamically Stable Gait:
Phase 1: Foot 1 heel roll contact, Foot 2 toe roll contact
Phase 2: Foot 1 flat contact, Foot 2 swinging freely
Phase 2: Foot 1 toe roll contact, Foot 2 swinging freely

B. Dynamic model of humanoid robot

Depicted in Fig. 2 is our current humanoid prototype. The humanoid construction consists of:

1) two legs each with 6 links and 6 actuated joints
2) hip has 3 DoF, knee 1 DoF, ankle 2 DoF
3) waist joint providing a rotation about vertical axis
4) each shoulder with 2 DoF
5) head is (temporarily) fixed to the body

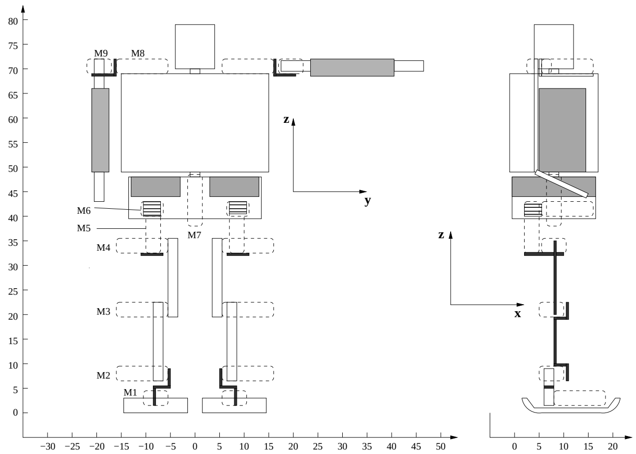

Fig. 2. Humanoid Kinematic Structure

The humanoid dynamic model consists of:

1) 17 *degrees of freedom* (DoF)
2) free-floating body with central reference point in the torso and a fictitional 6 DoF joint between it and an inertial reference frame
3) modeled as a tree structured multibody system (contacts are "cut" between robot and ground)

A total of 23 position and 23 velocity states $(\mathbf{q}(t), \dot{\mathbf{q}}(t))$ resulting in 46 differential equations describe the system configuration. Six position and 6 velocity states correspond to the fictional 6-DoF hinge between the inertial system and the main reference body. During the biped swing phase with one leg in contact, 34 independent states in a reduced dynamics model describe the system.

The humanoid inertia parameters are estimated by approximating each piece of the construction with a shape primitive (cylinder, ellipsoid, box) and individually weighing these pieces. The hip flexion/extension joint performing most of the work in the hip was placed last of the three hip joints; thus, the needless work of swinging the other two hip joints is saved. On the other hand, the flexion ankle joint is placed higher than the abduction joint so that at collision of the heel with the ground the impulsive force will disperse better throughout the body rather than influence primarily only the ankle joints.

III. Dynamic Stability and Performance

A. Measures for stable locomotion

The difficult task of maintaining stability of fast legged locomotion has been a main obstacle in the construction of such systems. The notion of *static stability*, often used to enforce *postural stability*, does not suffice for fast motion. Static stability requires the ground projected center of mass (GCoM) to lie within the *support polygon*, the convex hull about the leg's contact points. This highly conservative measure generally results in very slow legged motions. The notion of *dynamic stability* is required for faster legged motion. A dynamically stable gait is one without static stability that is sustainable indefinitely [2]; however, an adequate measure suitable for gait generation and control design is not currently available.

The center-of-pressure (CoP), equivalent to the zero-moment-point (ZMP), is a point on the ground where the net vertical ground reaction force acts. This point has often been used in previous research efforts [6] to provide a dynamic measure for postural stability by computing its distance to the support polygon boundary. Instability occurs when the CoP reaches the boundary, then a change in the system's contact condition generally occurs, and the system will begin to rotate about that edge. The COP's deficiency is that it always remains within the contact polygon, even during periods of instability and does not provide information as to neither the degree nor the direction of postural instability.

The foot-rotation-indicator (FRI) [2], the point on the ground where the net vertical contact force would have to act to keep the foot stationary, is more informative than the CoP. It coincides with the CoP when under the foot's contact surface. When not under the surface, its location gives information about the degree and direction of postural instability. When multiple feet are in contact, individual foot instabilities are indicated by the FRI, though not by the system-wide net CoP should they occur within the support polygon. In the moment a foot's FRI exits its ground contact surface, regardless of whether it continues in the support polygon, the foot changes its contact condition and rotates about that edge thus changing the system's dynamic behavior.

The FRI is calculated as follows. It is assumed that contact forces $\mathbf{f_c}$ cannot be measured with sensors and must be deduced from a dynamic equilibrium equation,

$$\mathbf{f_c} = -(J_c \mathcal{M}^{-1} J_c^T)^{-1} Q\dot{V}_c \,,$$

where $Q\dot{V}_c$ represents the accelerations of the unconstrained system along the constrained motion DoF at some reference point $\mathbf{p_r}$ on the foot's contact surface. In the case of flat foot contact, 6 motion DoF are constrained (3 linear and 3 rotational) so that $\mathbf{f_c} = [N_x\, N_y\, N_z\, F_x\, F_y\, F_z]^T$. The FRI point $\mathbf{p_f}$ is the point where an equivalent force $\mathbf{f_c}$ may be applied on the foot and for which $\{N_x = 0, N_y = 0\}$. This can be calculated from the spatial transformation of a force acting on a rigid body. Let $\mathbf{p_r} = [p_{r,x}\ p_{r,y}\ 0]^T$.

$$\mathbf{p_f} = [p_{r,x} - N_y/F_z \quad p_{r,y} + N_x/F_z \quad 0]^T \,. \qquad (3)$$

This point is calculated for each foot and constrained to lie inside the foot contact surface during an optimization. Its distance to a central position may also be minimized in the performance for optimal dynamic postural stability.

B. Optimization of stability and performance indices

Algebraic control strategies for legged systems cannot yet be constructed to handle the high dimension and many modeling constraints present in the locomotion problem. Heuristic control methods, on the other hand, tend to have poor performance with respect to power efficiency and stability. The remaining proven approach is the use of sophisticated numerical optimization schemes which can incorporate the numerous modeling constraints to generate optimal trajectories. The resulting trajectories may later be tracked or used to approximate a feedback controller in the portion of state space of interest. We list here three performance indices currently used in our humanoid gait generation investigations.

Postural Stability Performance: Distance in the ground plane between foot i's FRI point ${}_i\mathbf{p_f}$ and a central reference point under the foot ${}_i\mathbf{p_r}$

$$\mathbf{J}_{s1}[\mathbf{q}, \dot{\mathbf{q}}, \mathbf{u}] = \int_0^{t_f} \sum_i \left(\frac{{}_iN_x^2 + {}_iN_y^2}{{}_iF_z^2} \right) \mathrm{d}t \qquad (4)$$

where $\frac{{}_iN_x^2+{}_iN_y^2}{{}_iF_z^2} = ({}_ip_{f,x} - {}_ip_{r,x})^2 + ({}_ip_{f,y} - {}_ip_{r,y})^2$.

Energy Performance: In legged systems where a high torque is generated by a large current in the motor, the primary form of energy loss is called the Joule thermal loss [7]. The integral of this value over a gait period is

$$\mathbf{J}_{e1}[\mathbf{u}] = \frac{1}{s} \int_0^{t_f} \sum_{i=1}^{N} R_i \left(\frac{u_i}{G_i K_i} \right)^2 \mathrm{d}t \qquad (5)$$

where R_i, G_i, K_i, and u_i are the armature resistance, gear ratio, torque factor, and applied torque for link i respectively, while s is the step length or total distance traveled over one stride.

Efficiency Performance: The specific resistance ϵ as used in [3] measures the output power in relation to the mass moved and the velocity attained and is a dimensionless quantity. It represents a normalized form of the required kinetic energy

$$\mathbf{J}_{e2}[\dot{\mathbf{q}}, \mathbf{u}] = \int_0^{t_f} \frac{\sum_{i=1}^{N} |u_i \dot{q}_i|}{mgv} , \qquad (6)$$

where mg is the weight of the system, $\dot{q}_i$ is the joint i angle velocity and v is the average forward velocity.

The availability of a fully validated dynamic model combined with optimization tools permits one to make conclusive investigations into which stability or efficiency measures are most effective, though no one measure is sufficient for gait generation. The stability performance (4) cannot be used alone to verify or design a dynamically stable control strategy and must be combined with additional dynamic system measures. Efficiency is secondary in importance to stability in legged systems, but it can also have a strong influence in the successful design of an autonomous biped. A challenge for systems with limited power supply is to combine energy conserving motion with the robust, stability properties discussed previously.

Numerical optimization tools have advanced sufficiently [9] such that the many modeling and stability constraints can be incorporated into the problem formulation together with a relatively complete dynamical model so as to obtain truly realistic energy-efficient, stable and fast motions. The optimization approach is based on a discretization of the control problem in time using direct collocation and its subsequent formulation as a nonlinear programming problem (NLP) then solved with a sparse sequential quadratic programming algorithm.

The optimization of the stability or energy performance indices subject to the system dynamics and constraints leads to optimal control problems. Their solution delivers optimal open loop trajectories $\mathbf{x}^*(t)$, $\mathbf{u}^*(t)$, $0 \le t \le t_f$. The program DIRCOL [9] uses the method of sparse direct collocation and approximates the states $\mathbf{x}$ with spline functions, the controls $\mathbf{u}$ with linear functions, and constant parameters $\mathbf{p}$ on a discrete time grid. The method is equipped to handle the complexities of the walking problem: unknown liftoff times, different ground contact combinations for the legs, discontinuous states at collision times of the legs with the ground, switching dynamics, and actuation limits.

IV. Results for Autonomous Biped Design and Dynamics of Locomotion

A. Design considerations of biped walking machine

One is faced with a difficult compromise in the design of an autonomous biped. Maximum agility and speed of locomotion require strong motors and gears. The actuators, though, must be as light as possible for autonomous operation and without extensive power consumption leading to heavy on-board batteries. A strategy for finding a good compromise between these conflicting goals using dynamic optimization was presented in [10]. The resulting architecture of the 80 cm humanoid robot is shown in Fig. 2.

An energy performance optimization criterion (5) was investigated subject to a rigid body dynamics model (1) and maximum input power constraints. The maximum output wattage M_W in the power constraints was first selected to determine the motor class.

$$\max_{t \in [0, t_f], i \in \{1,\ldots,n\}} |\dot{q}_i(t) u_i(t)| \le M_W \qquad (7)$$

Consequently, optimal trajectories provide information as to the maximum required torques, joint velocities and accelerations in order to produce a walking gait at a specified forward velocity.

The optimization problem was solved for our humanoid prototype for various mass configurations. The maximum torque requirements over all joints are plotted against various motor characteristic lines with different nominal supply voltages and repeated for different gear ratios.

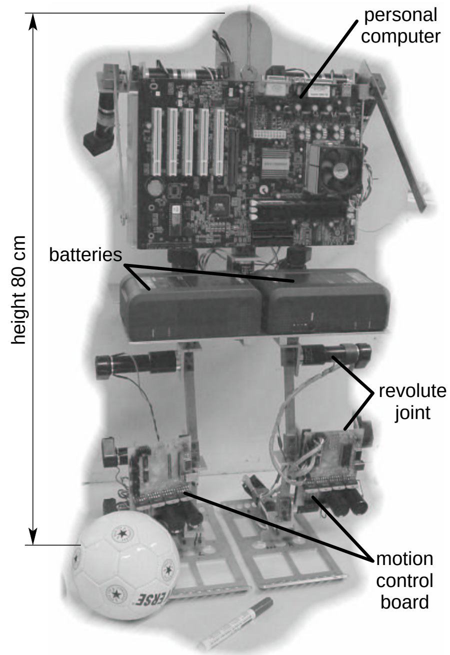

Fig. 3. Mechanical Realization of the First Biped Prototype

The motor-gear combination was selected which provided the maximum torque and velocity combinations in the calculated workspace. Our investigations led to a 42V motor with a 66:1 gear ratio.

B. Selection and integration of fundamental hard- and software

The mechanical biped construction (cf. Fig. 3) is based on linking elementary modules each consisting of motor, gear, pulse encoder, L-shaped base plate, and lever arm [10]. This prototype carries three batteries for the power supply of the motors, two of them visible on the picture at the height of the hips and below the waist joint. The third battery is located symmetrically behind the hips. The chosen Sony BP-L90A batteries provide a capacity of 90 Wh each, hence allowing for approximately 45 min autonomous walking.

For this protoype, a standard ATX mainboard with Athlon 1300 MHz CPU has been chosen providing enough computational power for motion control and additional tasks such as object recognition using a camera system. Its power is supplied by two Bebop Endura E-50S batteries.

The motors are accessed using an USB motion control board developed at the Control Systems Group in Berlin [10]. It consists of an 8 bit microcontroller including 3 USB endpoints, a 6 channel A/D converter and a 16 channel pulse-width modulator (PWM) admitting a motor load of up to 3 A at 55 V. The actual motor position is determined by evaluating the signals of pulse encoders attached to each motor. Up to 4 motors can be connected to each board weighing 170 g. In consideration of the USB

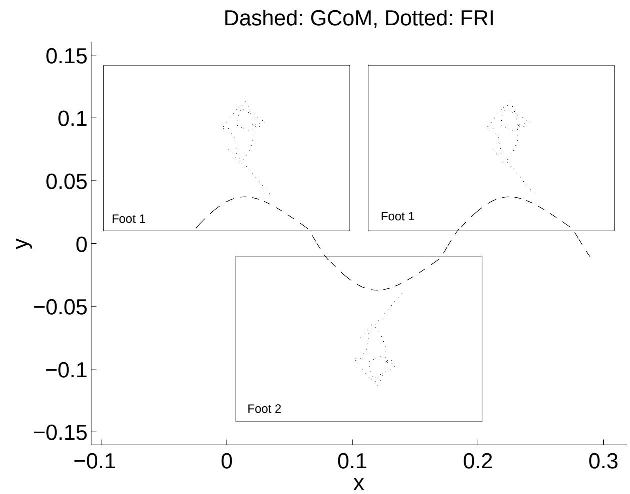

Fig. 4. GCoM and individual foot FRI trajectories during two steps of an optimized statically stable walk.

control transfer mode, mean USB communication delay as well as microcontroller computational times required by USB service routines and PD control routines, PD control loops and communication runs have been designed and implemented at 250 Hz giving satisfactory control performance for this prototype.

The current joint angles are sensed by pulse encoders. The moments acting at each joint may be computed from the sensed motor currents. Three gyroscopes and a 3-dimensional accelerometer are filtered using a Kalman filter for the inertial estimation of the main reference body.

A graphical user interface has been developed for rapid control prototyping. The MATLAB Realtime Workshop provides a solution by permitting the generation and compilation of standalone real-time code for RT Linux from a SIMULINK model. The control loop to be implemented is composed of ordinary SIMULINK blocksets, hence the migration from designing a controller in offline mode to evaluating it in an experiment is subject to substituting the system model by hardware in the loop which can also be accessed through SIMULINK blocksets.

C. Computation of humanoid reference trajectories

Inverse kinematic algorithms that were developed for the humanoid prototype in order to compute the reduced dynamics (2) also facilitated the generation of heuristic joint angle and angle velocity reference trajectories satisfying the physical modeling constraints (Sect. II-A). The reference trajectories served as start trajectories for the complex 3-dimensional humanoid gait optimization.

Several stages of gait optimizations were performed with varying complexity until all physical and stability constraints were included in the 3-D optimizations. An energy performance index was chosen (5) subject to the statically stable and dynamic postural stability nonlinear

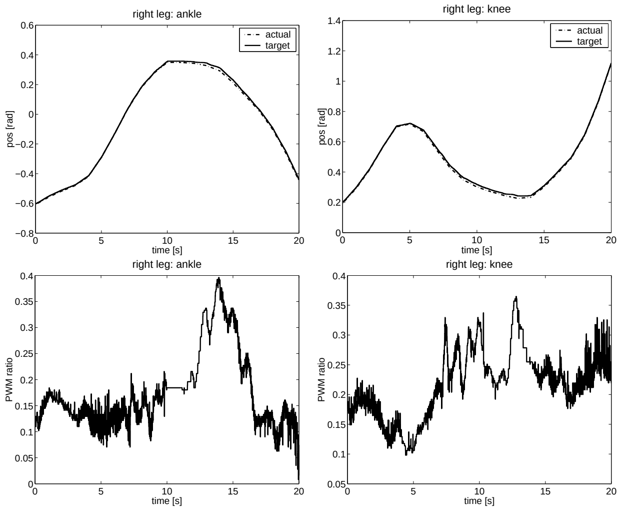

Fig. 5. First Experiments with Trajectory Following Control (left: ankle joint, right: knee joint of right leg)

constraints (Sect. III-A). First investigations using a dynamic model considering only the 12 joint DoF in the legs were made using statically stable gaits, walking on flat feet, with one swing phase composing 80–85% of the gait period and a double contact phase composing the remainder of the gait period. Thus a 25-dimensional ODE (including the objective) has been optimized subject to numerous explicit and implicit nonlinear boundary constraints and nonlinear inequality constraints (Sect. II-A. An optimization using 44 time grid points required 1584 NLP variables (Sect. III-B) with 1079 nonlinear equality constraints and 220 nonlinear inequality constraints. The necessary run-time after two automatic grid refinements using a reasonable starting solution was 1418 seconds on a Pentium III, 1150 MHz. The GCoM and individual foot FRI trajectories from the optimal gait are displayed in Fig. 4. Note that the system remains statically stable and that the FRI points remain centered about the middle of their respective foot contact surfaces.

D. First walking experiments for biped prototype

First walking experiments for statically stable leg motions are displayed in Fig. 5 where trajectory following control has been performed with a position error of less than 0.019 rad. The displayed PWM ratio signals are less than 0.5. Thus, the utilized DC motors offer enough power for faster gaits.

V. Conclusions and Outlook

The development of autonomous bipedal robots presents the particularly difficult challenge of first constructing a mechanism powerful and light enough to propel its own weight forward while taking steps. It must also operate efficiently enough not to require an excessive battery supply thereby further increasing its own weight. We address these challenges in this paper by presenting a set of techniques for modeling, simulation and optimization of complete dynamic models of the legged system. The combination of efficient reduced dynamics algorithms with direct collocation methods and nonlinear optimization software is first used to deduce the best motor/gear combination in the construction of a humanoid prototype. Then optimal gait trajectories are calculated for a precise dynamic model with 24 states for 3-dimensional motion of the humanoid, one of the highest dimensional optimizations as of yet completed for a legged system. Stability, energy and efficiency numerical performance measures are presented, discussed and implemented. Finally, the control implementation and experimental trajectory tracking results are presented.

VI. References

[1] Featherstone, R.; Orin, D.: “Robot Dynamics: Equations and Algorithms.” IEEE Intern. Conf. on Robotics and Automation (2000) 826–34

[2] Goswami, A.: Postural Stability of Biped Robots and the Foot-Rotation Indicator (FRI) Point. Intern. J. of Robotics Research **18**(6) (1999) 523–533

[3] Gregorio, P.; Ahmadi, M.; Buehler, M.: “Design, Control, and Energetics of an Electrically Actuated Legged Robot.” IEEE Transactions on Systems, Man and Cybernetics, Part B **27**(4) (1997) 626–634

[4] M. Hardt, O. von Stryk: “Dynamic modeling in the simulation, optimization and control of legged robots.” To appear in: Zeitschrift für Angewandte Mathematik und Mechanik (2003)

[5] K. Hirai, M. Hirose, Y. Haikawa, and T. Takenaka, “The Development of Honda Humanoid Robot,” *IEEE International Conference on Robotics and Automation* (1998) 1321–1326

[6] Huang, Q.; Yokoi, K.; Kajita, S.; Kaneko, K.; Arai, H.; Koyachi, N.; Tanie, K.: “Planning Walking Patterns for a Biped Robot.” IEEE Transactions on Robotics and Automation **116** (1994) 30–36

[7] Kimura, H.; Shimoyama, I.; Miura, H.: “Dynamics in the Dynamic Walk of a Quadruped Robot.” Advanced Robotics **4**(3) (1990) 283–301

[8] Jain, A.,; Rodriguez, G.: “Computational Robot Dynamics Using Spatial Operators.” IEEE Intern. Conf. on Robotics and Automation (2000) 843–49

[9] O. von Stryk “User’s Guide for DIRCOL Version 2.1,” Simulation and Systems Optimization Group, Technische Universität Darmstadt, http://www.sim.informatik.tu-darmstadt.de/sw/dircol

[10] D. Wollherr, M. Hardt, M. Buss, O. von Stryk, “Actuator selection and hardware realization of a small and fast-moving autonomous humanoid robot,” in *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS)* (2002) 2491-2496