Optimum Design of Microwave Oscillators with Minimized Phase Noise

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1 Introduction

A low phase noise is besides the signal properties essential to the design of oscillating electrical circuits. Up to now, a reduction of the phase noise can only be achieved by using empirical rules which, e. g., require additional elements in the circuit or the manufacture of lines of prototypes (trial and error).

A new and general method to minimize the single-sideband phase noise of free running oscillators is presented. It is based on the description of the signal and noise behavior of an oscillator circuit by the Langevin equations

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t)) \\ &= \mathbf{f}(\mathbf{x}(t)) + \mathbf{G}(\mathbf{x}(t)) \cdot \boldsymbol{\xi}(t) + \mathbf{g}(\mathbf{x}(t)) \cdot y(t) + \mathcal{O}(\boldsymbol{\xi}^2, y^2, \boldsymbol{\xi} y), \quad (1.1) \\ \mathbf{G}(\mathbf{x}(t)) &= \frac{\partial \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t))}{\partial \boldsymbol{\xi}} |_{\boldsymbol{\xi}=0, y=0}, \\ \mathbf{g}(\mathbf{x}(t)) &= \frac{\partial \mathbf{f}(\mathbf{x}(t), \boldsymbol{\xi}(t), y(t))}{\partial y} |_{\boldsymbol{\xi}=0, y=0}, \end{aligned}$$

where $\mathbf{x} = (x_1, \ldots, x_n)^T$ are the state variables of the circuit (voltages, currencies), $\boldsymbol{\xi}$ are the white noise sources and y is a nonlinear $f^{-\alpha}$ noise source denoting the baseband noise. The noise sources are assumed to be small compared to the signal amplitudes and the terms of order $\mathcal{O}(\boldsymbol{\xi}^2, y^2, \boldsymbol{\xi} y)$ are neglected.

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2 Single-Sideband Phase Noise

The single-sideband phase noise $L(f_m)$ of free running oscillators can be calculated by solving Eq. (1.1) and using a perturbation approach (Kärtner [5])

$$L(f_m) = \frac{\Delta f_{3dB}}{\pi f_m^2} + \omega_0^2 |g_{1,0}|^2 \frac{c}{|2\pi f_m|^{2+\alpha}} \tag{1.2}$$

$$\text{with} \quad \Delta f_{3dB} = \frac{1}{4\pi}\omega_0^2 \frac{1}{T_0}\int_0^{T_0} \mathbf{v}(\mathbf{x}(t))^T \mathbf{G}(\mathbf{x}(t)) \mathbf{\Gamma}(\mathbf{x}(t)) \mathbf{G}(\mathbf{x}(t))^T \mathbf{v}(\mathbf{x}(t)) \mathrm{d}t$$

$$\text{and} \quad g_{1,0} = \frac{1}{T_0}\int_0^{T_0} \mathbf{v}(\mathbf{x})^T \mathbf{g}(\mathbf{x}) \mathrm{d}t.$$

The first term on the right hand side of Eq. (1.2) describes the phase noise caused by the white noise sources. $\mathbf{v}(\mathbf{x}(t))$ is the left-sided eigenvector of the eigenvalue 1 of the fundamental matrix $\mathbf{\Psi}(T_0, 0)$. The matrix $\mathbf{\Gamma}$ denotes the correlation matrix of the white noise sources. The second term of Eq. (1.2) describes the phase noise caused by the baseband noise. The factor $g_{1,0}$ characterizes the upconversion of the baseband noise to the carrier frequency $\omega_0 = 2\pi/T_0$. The modulation of the $f^{-\alpha}$ noise source due to the oscillation is taken into account as well as the upconversion of the baseband noise caused by the nonlinearities in the circuit. The factor c is derived from baseband noise measurements. f_m is the offset frequency where the single-sideband phase noise is to be calculated. The functions $\mathbf{x}(t)$ and $\mathbf{v}(t)$ depend on the design parameters $\mathbf{p}$ of the circuit (e. g., capacitors and inductors of the linear network) by the system of $2n$ nonlinear differential equations derived from (1.1)

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{p}), \tag{1.3}$$

$$\dot{\mathbf{v}}(t) = -\left[\frac{\partial \mathbf{f}(\mathbf{x}(t), \mathbf{p})}{\partial \mathbf{x}}\right]^T \cdot \mathbf{v}(t), \quad t \in [0, T_0] \tag{1.4}$$

and $2n+1$ boundary conditions

$$r(x(0), v(0), x(T_0), v(T_0)) = 0. \tag{1.5}$$

3 Optimal Control Problem

The minimization of the phase noise $L(f_m)$ with respect to the design parameters $\mathbf{p}$ and subject to Eqs. (1.3) – (1.5) is an optimal control problem with **p** as (constant) control function from a finite dimensional control space and **x** and **v** as state variables of the optimal control problem. The optimization problem is solved numerically by the direct collocation method DIRCOL [9]. The method is based on a discretization of the state variables by piecewise cubic spline functions satisfying the dynamic equations at the grid points of a time grid and at the centers between (cf. Hargraves, Paris [4] and [8]). By this approach, the optimal control problem is transformed to a (finite dimensional) nonlinearly constrained optimization problem that can be solved by the Sequential Quadratic Programming method due to Gill et al. [2]. Compared with other methods for solving optimal control problems the direct collocation method is easy-to-use (as knowledge of optimal control theory is not required), robust (not much information on the solution is a priori required) and reliable (if accuracy requirements are not too high) [9].

4 Numerical and Experimental Results

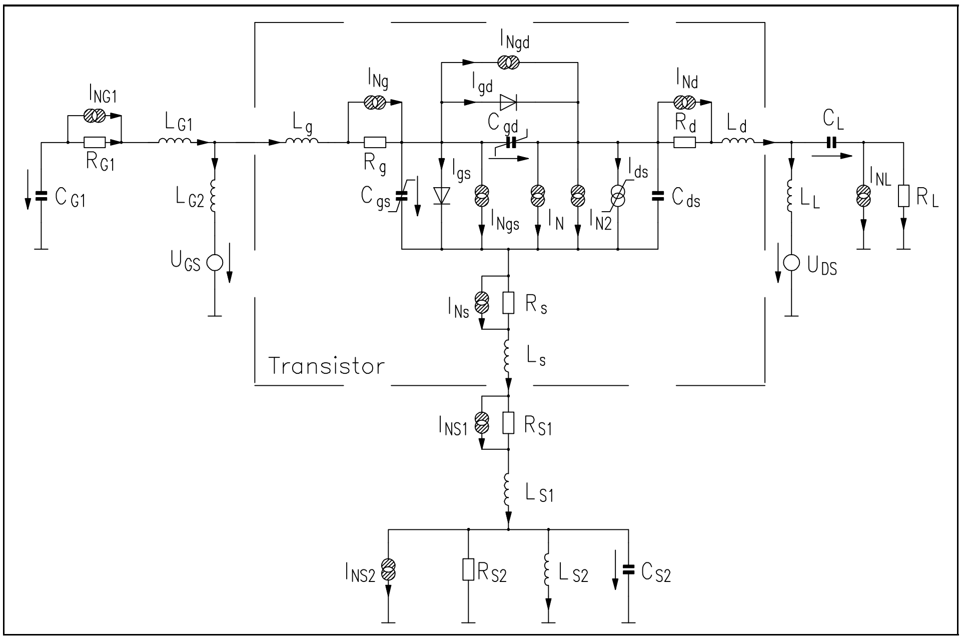

Fig. 1 Equivalent circuit of the oscillator to be optimized.

The new method to improve the design of oscillators is applied to minimize the single-sideband phase noise of a planar integrated [3] free running microwave oscillator at 15 GHz [1], [6]. A GaAs MESFET NE710 is used for the active element which is connected at the gate and source terminals with microstrip lines as resonators. The equivalent circuit of the oscillator is depicted in Fig. 1. In this special case, the five design parameters L_{G1}, C_{G1}, L_{S1}, L_{S2}, and C_{S2} of the linear network are optimized subject to a system of 20 highly nonlinear differential equations (1.3), (1.4). Only the upconverted baseband noise to the carrier frequency is taken into account to minimize the single-sideband phase noise $L(f_m)$ in Eq. (1.2). After the computation of the optimal values of the five design parameters a prototype MIN2 of the new designed oscillator has been manufactured [6] (Fig. 2).

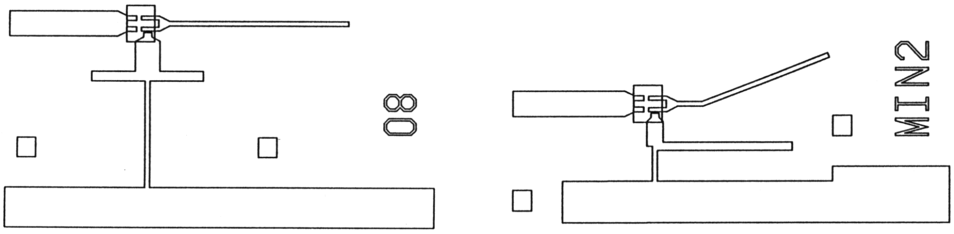

Fig. 2 Layout of the original oscillator O8 (left side) and of the optimized oscillator MIN2 (right side) with a scaling factor of about 10:1.

The measured output power of the reference oscillator O8 is 12.7 dBm compared with the output power of 14.4 dBm of the oscillator with the minimized phase noise. For the prototype MIN2 a reduction of 8 dB of the phase noise caused by the upconverted baseband noise is measured at a frequency deviation of 10 kHz compared to the original oscillator O8 (Fig. 3) [1], [6]. The difference of the measured (——) from the calculated single-sideband phase noise (– · – · –) of the optimized oscillator is mainly due to deviations in the technical data of the elements used to manufacture the prototype MIN2. The measured peaks between 400 kHz and 7 MHz are due to external interferences.

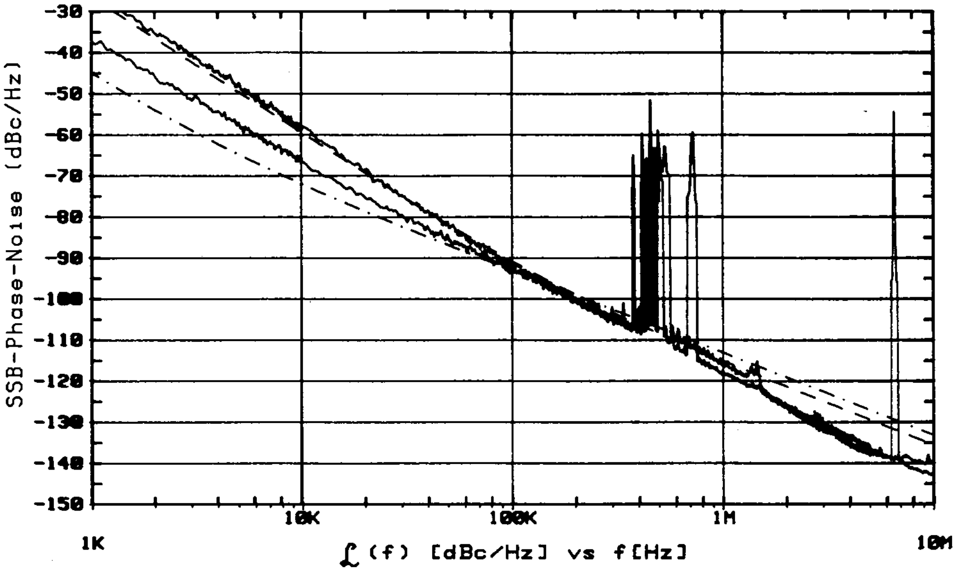

Fig. 3 Measured (——) and simulated single-sideband phase noise of the original oscillator (– – –) and the oscillator with minimized phase noise (–·–·–).

5 Conclusions

A new method for the design of microwave oscillators with minimized phase noise has been outlined and successfully applied to reduce the single-sideband phase noise of a certain high frequent oscillator. With the new method a significant reduction of the phase noise is achieved in both simulation and experiment. Hereby neither the use of additional elements in the circuit nor the manufacture of lines of prototypes in advance is required.

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