Sonderforschungsbereich 438

Mathematische Modellierung, Simulation und Verifikation
in materialorientierten Prozessen und intelligenten Systemen

Modelling and Simulation of Rheological Fluid Devices

Butz, Torsten; von Stryk, Oskar

Preprint SFB-438-9911
Mai 99

Technische Universität München    Universität Augsburg
Cataloging Data:

Butz, Torsten; von Stryk, Oskar: Modelling and Simulation of Rheological Fluid Devices; Sonderforschungsbereich 438: Technische Universität München, Universität Augsburg; Preprint SFB-438-9911(99)

Mathematics Subject Classification: 65C20; 76A05

Editor:

K.H. Hoffmann (hoffmann@appl-math.tu-muenchen.de),
Technische Universität München, D-80290 München, Germany.

For the Electronic Version see: http://www-lit.mathematik.tu-muenchen.de/reports/html/SFB/

Copyright © XCIX Sonderforschungsbereich 438, München, Augsburg. All rights reserved.
Printed in Germany.
Modelling and Simulation of
Rheological Fluid Devices

Torsten Butz¹ and Oskar von Stryk²

¹,² Lehrstuhl M2 Höhere Mathematik und Numerische Mathematik
Technische Universität München, D-80290 München,
World Wide Web: http://www-m2.ma.tum.de
¹ butzt@ma.tum.de ² stryk@ma.tum.de

Abstract

Electro- and magnetorheological fluids are smart, synthetic fluids changing
their viscosity from liquid to semi-solid state within milliseconds if a suffi-
ciently strong electric or magnetic field is applied. When used in suitable
devices, they offer the innovative potential of very fast, adaptively control-
able interfaces between mechanical devices and electronic control units. This
paper gives an overview on the basic properties of rheological fluids and dis-
cusses various phenomenological models for rheological fluid devices and their
applications. Numerical simulation results are presented for the passive sus-
pension of a quarter vehicle model.
## Contents

1 Introduction .......................................................... 1

2 ER and MR Mechanisms and Fluid Properties .................. 2

3 Phenomenological Models for Rheological Fluid Devices .... 4
   3.1 Parametric Models ............................................ 6
      3.1.1 Bingham Model ......................................... 6
      3.1.2 Extended Bingham Model .............................. 7
      3.1.3 Three Element Model (Powell 1994) .................. 8
      3.1.4 BingMax Model ......................................... 10
      3.1.5 Bouc-Wen Model ........................................ 11
      3.1.6 Modified Bouc-Wen Model ............................ 12
      3.1.7 Non-Linear Viscoelastic-Plastic Model
            (Kamath and Wereley 1997a) ......................... 14
      3.1.8 Augmented Non-Linear Viscoelastic-Plastic Model
            (Kamath and Wereley 1997b) .......................... 16
      3.1.9 Other Models ............................................ 18
   3.2 Non-Parametric Models ........................................ 19
      3.2.1 Chebyshev Polynomial Fit ............................. 19
      3.2.2 Neural Networks ........................................ 20

4 Numerical Simulation of a Passive Rheological Vibration
   Damper ........................................................... 21
   4.1 Quarter Vehicle Model ....................................... 22
   4.2 Bingham Model of a Passive Rheological Damper ......... 23
   4.3 Bouc-Wen Model of a Passive Rheological Damper ....... 23
   4.4 Simulation Results for a Step Disturbance ............... 24
   4.5 Simulation Results for a Sinusoidal Bump ............... 26

5 Further Applications of Rheological Fluid Devices .......... 30

6 Conclusions and Outlook ........................................... 30

References ............................................................ 31
1 Introduction

Electro- (ER) and magnetorheological (MR) fluids are colloidal suspensions which exhibit large reversible changes in flow properties such as the apparent viscosity when subjected to sufficiently strong electric and magnetic fields, respectively (Winslow 1949). They usually consist of micron-sized polarisable or magnetisable solid particles dissolved in a non-conducting liquid like mineral or silicone oil. In general, the composition of rheological fluids exhibits a broad diversity concerning solvent, solute and additives (see for example Block and Kelly 1988, Carlson and Spencer 1996).

In recent years, ER and MR fluids have attracted considerable interest due to their wide range of use in vibration dampers for vehicle suspension systems, machinery mounts or even seismic protection of structures; their stiffness and damping capabilities can be adjusted very quickly by applying a suitable electric or magnetic field (Stanway et al. 1996).

Rheological fluid dampers enable active and semi-active vibration control systems with reaction times in the range of milliseconds and, additionally, low power requirements when using MR fluids. Due to their rather simple mechanical design which involves only few moving parts they ensure high technical reliability and exhibit almost no wear. Thus, continuously adjustable rheological fluid devices offer the innovative potential of robust and fast controllable interfaces between mechanical components and electronic control units.

Scientific challenges in the field of rheological fluids and devices consist in:

1. The development of (optimal) control strategies for rheological fluid devices: Because of the intrinsically nonlinear nature of semi-active control devices, development of output feedback control strategies that are practically implementable and can fully utilize the capabilities of these unique devices is another important, yet challenging, task (Spencer 1996).

Though, to develop suitable control algorithms for electro- and magnetorheological fluid devices mathematical and physical models are needed that can accurately reproduce their non-linear behaviour.

2. The mathematical modelling and numerical simulation of rheological fluids and devices: Note: The mechanism of ER fluids is not
well understood and hence what is theoretically possible is not known (Hartsock, Nowak and Chaundy 1991, p. 1310).

The outline of this paper is as follows: After a description of ER and MR mechanisms and fluid properties several phenomenological models for rheological fluid devices are presented according to results of the current research. Their validity is discussed as far as comparisons with experimental data have been reported within the literature. Finally, different discrete element models are applied to a rheological vibration damper, and its operation is simulated numerically.

2 ER and MR Mechanisms and Fluid Properties

It is commonly agreed that the main mechanism of the ER response is based on some form of polarisation due to the dielectric mismatch between the suspended particles and the solvent (Powell 1994, Bonnecaze and Brady 1992a). A qualitative explanation accounts for the particles as dipoles. Accordingly, the electrostatic forces cause the formation of particle chains in the direction of the electric field. The tendency of columnar formation has been observed experimentally as well as the development of larger particle clusters with increasing particle concentration (Gast and Zukoski 1989, Klingenberg et al. 1989). However, Powell (1994) mentions another proposed mechanism of the induced polarisation of an ionic double layer.

Electrorheological fluids exhibit a yield phenomenon when subjected to a sufficiently large electric field. A yield stress increasing with the field strength must be overcome to start fluid flow (between stationary electrodes) or shearing of the fluid (between moving electrodes) perpendicular to the applied field. The exponents of the power law \( \tau_y = E^n \) relating the field strength \( E \) to the yield stress \( \tau_y \) are reported to range from 1.2 to 2.5 depending on the consistency of the suspension (Gavin et al. 1996a). In terms of the above described mechanisms the yield point may be related to the breakage of particle chains. In the pre-yield region ER fluids behave like elastic solids, resulting from chain stretching with some occasional rupture; the post-yield region reflects an equilibrium between chain rupture and reformation, the fluid exhibits viscous properties (Choi et al. 1990, Gamota and Filisko 1991). The structural processes, more precisely described by Powell (1994) and Kamath
et al. (1996), differ according to the respective operating modes (cf. Section 3, Figure 1), but obviously they are of minor significance for modelling rheological devices in general.

<table>
<thead>
<tr>
<th>Property</th>
<th>ER Fluid</th>
<th>MR Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>response time</td>
<td>milliseconds</td>
<td>milliseconds</td>
</tr>
<tr>
<td>plastic viscosity $\eta$</td>
<td>0.2 to 0.3 Pa\cdot s</td>
<td>0.2 to 0.3 Pa\cdot s</td>
</tr>
<tr>
<td>(at 25°C)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>operable temperature range</td>
<td>$+10$ to $+90\degree$ C (ionic, DC)</td>
<td>$-40$ to $150\degree$ C</td>
</tr>
<tr>
<td></td>
<td>$-25$ to $+125\degree$ C (non-ionic, AC)</td>
<td></td>
</tr>
<tr>
<td>power supply (typical)</td>
<td>2 to 5 kV</td>
<td>2 to 25 V</td>
</tr>
<tr>
<td></td>
<td>1 to 10 mA</td>
<td>1 to 2 A</td>
</tr>
<tr>
<td></td>
<td>(2 to 50 watts)</td>
<td>(2 to 50 watts)</td>
</tr>
<tr>
<td>maximum yield stress $\tau_y$</td>
<td>2 to 5 kPa</td>
<td>50 to 100 kPa</td>
</tr>
<tr>
<td></td>
<td>(at 3 to 5kV/mm)</td>
<td>(at 150 to 250 kA/m)</td>
</tr>
<tr>
<td>maximum field</td>
<td>ca. 4 kV/mm</td>
<td>ca. 250 kA/m</td>
</tr>
<tr>
<td>$\eta/\tau_y^2$</td>
<td>$10^{-4}$ to $10^{-3}$ s/Pa</td>
<td>$10^{-10}$ to $10^{-14}$ s/Pa</td>
</tr>
<tr>
<td>density</td>
<td>1 to 2 g/cm$^3$</td>
<td>3 to 4 g/cm$^3$</td>
</tr>
</tbody>
</table>


Bonnecaze and Brady (1992a) proposed a molecular-dynamics-like simulation of an ER suspension subjected to both shear flow and an orthogonal electric field. The method allows for hydrodynamic, e.g., viscous, interparticle interactions determined by Stokesian dynamics as well as for electrostatic forces that are derived from the system electrostatic energy. The simulation is able to predict the apparent viscosity in accordance with the experimental data. Combined with an idealised chain model for the microstructure of the activated fluid it can further approximate the experimentally observed values for the yield stress (Bonnecaze and Brady 1992b). Except for small shear rates (expressed in terms of the so-called Mason number) the results of the simulation match with the Bingham plastic model described in the next section (Bonnecaze and Brady 1992a). Recently, Engelmann et al. (1999) proposed an extension of the classical Bingham model which goes beyond pure shear flows and enables the simulation of electrorheological fluids in complex geometries.

In the case of magnetorheological fluids a magnetic field causes the chain-like arrangement of the suspended particles by inducing a magnetic moment.
In addition, MR fluids exhibit a yield stress increasing with the applied field, and both a pre-yield region, characterised by elastic properties, and a post-yield region, characterised by viscous properties (Jolly et al. 1996). Due to their qualitatively similar behaviour phenomenological models of ER and MR fluid devices can mostly be applied to either material (Kamath and Wereley 1996).

Some properties of typical electro- and magnetorheological fluids are provided in Table 1. Exhibiting about the same response time and plastic viscosity $\eta$ as their ER analogues, MR fluids are less sensitive to impurities, such as water, usually occurring during manufacturing and usage. They have a larger operating temperature range, and they can be controlled with a considerably lower voltage (Spencer et al. 1997). Moreover, the yield stress achievable with MR suspensions is at least an order of magnitude greater just as the material property $\eta/\tau_0^2$, so that MR devices only require a comparatively small amount of fluid and space (Weiss et al. 1993, Spencer 1996). However, since often iron is used as a solute, the density of MR fluids is significantly higher than for typical ER suspensions. In addition, a greater variety of materials is available for ER fluids, and electric fields are often more suitable for complex geometries and small dimensions (Böse 1998).

3 Phenomenological Models for Rheological Fluid Devices

To take maximum advantage of electro- and magnetorheological fluids in control applications a reliable method is needed to predict their non-linear response. Several phenomenological models characterising the behaviour of rheological fluid devices have been presented. These models concern ER test devices (Gamota and Filisko 1991, Ehrrott and Masri 1992, Powell 1994, Gavin et al. 1996a,b, Kamath and Wereley 1997a) as well as ER and MR dampers for both seismic protection (Makris et al. 1996a,b, Burton et al. 1996, Spencer et al. 1997, Dyke et al. 1997) and vehicle applications (Stanway et al. 1987, Kamath et al. 1996, Kamath and Wereley 1996, 1997b). But despite the promise of controllable fluids most of the current models solely reproduce the rheological behaviour for constant field strengths. So far few concepts have been developed that allow for varying field strengths as needed for the design of active control strategies.
Most often, the considered devices operate in the valve (flow) mode, the direct shear mode or a combination of the two modes (Stanway et al. 1996). In a control valve the electrorheological fluid is constrained between stationary electrodes, and its resistance to flow is accommodated by adjusting the applied field (Figure 1a); frequently the performance of hydraulic piston/cylinder dampers is controlled in this way. In the latter mode, the fluid is subjected to direct shear between electrodes translating or rotating perpendicular to the field (Figure 1b). Few attention is paid to rheological devices employing the most recently introduced mode of squeeze-flow. This operating mode involves electrode motion in the direction of the applied field (Figure 1c); therefore the field strength continually varies according to the electrode distance (Stanway et al. 1996). In case of magnetorheological fluids the magnetic poles take the place of the electrodes.

![Figure 1](image_url)

Figure 1. Basic operating modes of rheological fluid devices.
(a) Valve mode. (b) Direct shear mode. (c) Squeeze-flow mode.

The proposed models do not strictly distinguish between the different modes of operation; but commonly adjustments have to be made corresponding to the design and the components of the respective devices. For reasons of wider commercial availability the recent research has mainly concentrated on modelling the ER fluid response (Kamath and Wereley 1997b). But due to their similar behaviour little difference is made between electro- and magnetorheological materials in the following survey. It is useful to distinguish between models which qualitatively simulate the rheological response and are fitted to experimental results by adjusting few parameters (parametric models) and models which are entirely based on the performance of a specific fluid device (non-parametric models).
3.1 Parametric Models

Parametric models are represented by a mathematical function whose coefficients are determined rheologically, i.e., the parameter values are adjusted until the quantitative results of the model closely match the experimental data. Thus, the dynamic response of rheological devices is reproduced by a semi-empirical relationship. Numerous parametric models can easily be described by an arrangement of mechanical elements such as springs and viscous dashpots.

3.1.1 Bingham Model

Most commonly the behaviour of rheological fluids is described by the Bingham plastic model. An ideal Bingham body behaves as a solid until a minimum yield stress $\tau_y$ is exceeded and then exhibits a linear relation between the stress and the rate of shear or deformation. Accordingly the shear stress $\tau$ developed in the fluid is given by

$$\tau = \tau_y \cdot \text{sgn}(\dot{\gamma}) + \eta \dot{\gamma}$$  \hspace{1cm} (1)

where $\dot{\gamma}$ is the (shear) strain rate and $\eta$ denotes the plastic viscosity of the fluid, i.e., the (Newtonian) viscosity at zero field (Gavin et al. 1996a).

![Bingham model](image)

Figure 2. Bingham model (Spencer et al. 1997).

In order to characterise the ER damping mechanism Stanway et al. (1987) proposed a mechanical model, commonly referred to as the Bingham model, that combines viscous and Coulomb friction. The mechanical analogue, a Coulomb friction element in parallel with a viscous dashpot, is shown in Figure 2. In this model, the force $F$ generated by the rheological device is given by

$$F = f_c \cdot \text{sgn}(\dot{x}) + c_0 \ddot{x}$$  \hspace{1cm} (2)
where \( \dot{x} \) denotes the velocity attributed to the external excitation, and where the damping coefficient \( c_0 \) and the frictional force \( f_c \) are related to the fluid’s viscosity and the field dependent yield stress respectively (Spencer et al. 1997).

The Bingham model accounts for electro- and magnetorheological fluid behaviour beyond the yield point, i.e., for fully developed fluid flow or sufficiently high shear rates. However, it assumes that the fluid remains rigid in the pre-yield region. Thus, the Bingham model does not describe the fluid’s elastic properties at small deformations and low shear rates which is necessary for dynamic applications (Kamath and Wereley 1997a). A comparison between the predicted force-velocity characteristic and the result of experiments conducted by Spencer et al. (1997) is provided in Figure 3.

![Force vs Velocity Graph](image)

**Figure 3.** Comparison between the predicted (——) and the experimentally obtained (—) force-velocity characteristic for the Bingham model (Spencer et al. 1997).

### 3.1.2 Extended Bingham Model

Gamota and Filisko (1991) presented an extension of the Bingham model to describe the ER fluid behaviour in the pre-yield and the post-yield region as well as at the yield point. This viscoelastic-plastic model consists of the Bingham model in series with the three-parameter element of a linear solid (Zener element) as shown in Figure 4. The force in this system is given by

\[
F = \begin{cases} 
  c_0 \dot{x}_1 + f_c \cdot \text{sgn}(\dot{x}_1) & , \ |F| > f_c \\
  k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1) + k_2(x_3 - x_2) & , \ |F| \leq f_c
\end{cases}
\] (3)
where again the damping coefficient $c_0$ and the frictional force $f_c$ in the Bingham model account for the plastic viscosity and the yield stress respectively. The field dependent parameters $c_1$, $k_1$ and $k_2$ are associated with the fluid's elastic properties in the pre-yield region (Gamota and Filisko 1991, Spencer et al. 1997).

![Extended Bingham model](image)

Figure 4. Extended Bingham model (Spencer et al. 1997).

As depicted in Figure 5, the extended Bingham model qualitatively describes the hysteretic response of the MR fluid device considered by Spencer et al. (1997). However, the resulting system of ordinary differential equations is extremely stiff due to the non-linear Coulomb friction element. Thus, the numerical simulation with explicit integration methods requires very small time steps (cf. Stoer and Bulirsch 1993).

![Force vs. Velocity](image)

Figure 5. Comparison between the predicted (——) and the experimentally obtained (-----) force-velocity characteristic for the extended Bingham model (Spencer et al. 1997).

### 3.1.3 Three Element Model (Powell 1994)

Focussing on predicting the behaviour of an ER fluid device Powell (1994) proposed a mechanical analogue consisting of a viscous damper, a non-linear
spring and a frictional element in parallel (Figure 6). In order to reproduce the hysteretic force-velocity characteristic that is observed experimentally (cf. Figure 7b), the Coulomb friction force \( f_c \) is modelled with static and dynamic friction coefficients \( f_{cs} \) and \( f_{cd} \) respectively. Furthermore, to facilitate the numerical integration smoothing functions are introduced for the friction force instead of the signum function. Hence,

\[
f_c = \begin{cases} 
 f_{cs} \left( 1 + \left( \frac{f_{cd}}{f_{cs}} \right) \cdot \exp(-a \, |\dot{z}|) \right) \cdot \tanh(e \ddot{z}) , & \dot{z} \cdot \ddot{z} \geq 0 \\
 f_{cd} \left( 1 - \exp(-b \, |\dot{z}|) \right) \cdot \tanh(e \ddot{z}) , & \dot{z} \cdot \ddot{z} < 0 
\end{cases}
\]  

(4)

where \( z \) is the displacement transmitted to the rheological device, and \( \dot{z} \) and \( \ddot{z} \) denote the corresponding velocity and acceleration respectively.

![Figure 6. Three element model (Powell 1994).](image)

The force generated by the device is given by

\[
F = f_c + f_k + c\ddot{z}
\]

(5)

where \( f_k = k \cdot \tanh(dz) \) is the non-linear force of a softening spring. In this model, the field-dependent values of the damping parameters \( f_{cs}, f_{cd}, a, b, c \) and \( e \) as well as the elastic parameters \( d \) and \( k \) are fitted to the experimental results. A comparison between the predicted and the observed behaviour of the ER device is provided in Figure 7.
3.1.4 BingMax Model

A discrete element model with similar components, referred to as the BingMax model, is reviewed by Makris et al. (1996b). It consists of a Maxwell element in parallel with a Coulomb friction element as depicted in Figure 8.

![Figure 8. BingMax model (Makris et al. 1996b).](image)

The force $F(t)$ in this system is given by

$$F(t) = K \int_0^t \exp \left( -\frac{t - \tau}{\lambda} \right) \dot{u}(\tau) \, d\tau + F_y \cdot \text{sgn} [\dot{u}(t)]$$  \hspace{1cm} (6)
where \( \lambda = \frac{C}{K} \) is the quotient of the damping constant \( C \) and the spring stiffness \( K \), and \( F_y \) denotes the permanent friction force. Equivalently the constituting Equation (6) can be written as

\[
F(t) + \lambda \cdot \frac{dF(t)}{dt} = C \cdot \dot{u}(t) + F_y \cdot \text{sgn}[-\dot{u}(t)]
\]

(7)

(see for example Bird et al. 1987).

To evaluate the performance of the BingMax model the predicted behaviour of the electrorheological device was compared with its experimental response to an earthquake excitation. The model is analysed in more detail by Burton (1996).

### 3.1.5 Bouc-Wen Model

In their survey of phenomenological models Spencer et al. (1997) presented the so-called Bouc-Wen model in order to characterise the behaviour of a MR fluid damper. This concept is based on an approach due to Wen (1976). It is supposed to reproduce the response of hysteretic systems to random excitations.

A mechanical analogue of the Bouc-Wen model is shown in Figure 9. The force generated by the device is given by

\[
F = c_0 \dot{x} + k_0 (x - x_0) + \alpha z
\]

(8)

where the hysteretic component \( z \) satisfies

\[
\ddot{z} = -\gamma |\dot{x}| z |z|^{n-1} - \beta \dot{z} |z|^n + \delta \dot{x}.
\]

(9)

By adjusting the parameter values \( \alpha, \beta, \gamma, \delta \) and \( n \) it is possible to control the shape of the force-velocity characteristic; an initial displacement \( x_0 \) of the spring was incorporated into the model to allow for the presence of an accumulator in the considered damper.

![Figure 9. Bouc-Wen model (Spencer et al. 1997).](image)
The Bouc-Wen model is well suited for the numerical simulation, since it is less stiff than the extended Bingham model. But as it is depicted in Figure 10, it cannot reproduce the experimentally observed roll-off in the yield region, i.e., for velocities with a small absolute value and an operational sign opposite to the sign of the acceleration.

![Force vs Velocity Graph](image)

Figure 10. Comparison between the predicted (——) and the experimentally obtained (-----) force-velocity characteristic for the Bouc-Wen model (Spencer et al. 1997).

### 3.1.6 Modified Bouc-Wen Model

To better predict the response of the MR damper in the region of the yield point Spencer et al. (1997) proposed an extension of the Bouc-Wen model which is depicted in Figure 11. The equations for the force in this system are given by

\[
F = \alpha z + c_0 (\dot{x} - \dot{y}) + k_0 (x - y) + k_1 (x - x_0) \\
= c_1 \dot{y} + k_1 (x - x_0)
\]  

(10)

where

\[
\dot{z} = -\gamma |\dot{x} - \dot{y}| |z|^{n-1} - \beta (\dot{x} - \dot{y}) |z|^n + \delta (\dot{x} - \dot{y})
\]

(11)

and

\[
\dot{y} = \frac{1}{c_0 + c_1} [\alpha z + c_0 \dot{x} + k_0 (x - y)].
\]

(12)

The hysteretic component z accounts for the time history of the response (Dyke et al. 1997). The spring \(k_1\) and its initial displacement \(x_0\) allow for both the additional stiffness and the force offset produced by the presence of an accumulator. The latter was included into the considered damper for reasons of pressure compensation.
To obtain a model which is valid for varying magnetic field strengths the parameters are assumed to depend on the voltage $v$ applied to the current driver as follows

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u$$  \hspace{1cm} (13)

$$c_1 = c_1(u) = c_{1a} + c_{1b} u$$  \hspace{1cm} (14)

$$c_0 = c_0(u) = c_{0a} + c_{0b} u$$  \hspace{1cm} (15)

where $u$ is governed by

$$\dot{u} = -\eta(u - v).$$  \hspace{1cm} (16)

The first order filter given by Equation (16) was introduced to allow for the fluid's dynamics of reaching rheological equilibrium.

A comparison between the force-velocity characteristic predicted by the modified Bouc-Wen model and the result of experiments conducted by Spencer et al. (1997) is provided in Figure 12. The model is able to accurately reproduce the MR fluid behaviour, even over a broad range of operating conditions (Spencer et al. 1997). Moreover, its parameter values are independent of the applied voltage and need not be estimated anew for different field strengths. However, the proposed model is highly dependent on the design and the components of the specific magnetorheological fluid device. In particular, an additional spring was incorporated to account for the accumulator present in the considered damper.
3.1.7 Non-Linear Viscoelastic-Plastic Model (Kamath and Wereley 1997a)

The viscoelastic-plastic model presented by Kamath and Wereley (1997a) combines two linear shear flow mechanisms with non-linear weighting functions in order to characterise the response of an ER fluid device.

Figure 13. Viscoelastic-plastic model (Kamath and Wereley 1997a).
(a) Viscoelastic mechanism. (b) Viscous mechanism.

In the pre-yield region the fluid’s behaviour is simulated by the three-parameter element of a linear fluid (Jeffreys model) as depicted in Figure 13a. The viscoelastic force \( F_{ve} \) generated by this system is governed by

\[
F_{ve} + \frac{C_1 + C_2}{K_1} \frac{dF_{ve}}{dt} = C_2 \dot{X} + \frac{C_1 C_2}{K_1} \dot{X}
\]

(17)

where \( C_1, C_2 \) and \( K_1 \) denote the parametric damping and stiffness constants respectively, and where \( X \) is the displacement transmitted to the device. In
the post-yield region the ER response is represented by the viscous relationship

\[ F_{vi} = C_v \dot{X} \]  

(18)

where the damping coefficient \( C_v \) is related to the apparent viscosity of the fluid (Figure 13b).

The transition from the pre-yield to the post-yield regime is performed by nonlinearly combining the viscoelastic and viscous components \( F_{ve} \) and \( F_{vi} \) to the net force

\[ F = F_{ve} S_{ve} + F_{vi} S_{vi}. \]  

(19)

The shape functions

\[ S_{ve} = \frac{1}{2} \left[ 1 - \tanh\left( \frac{\alpha - \alpha_y}{4 \varepsilon_y} \right) \right] \]  

(20)

and

\[ S_{vi} = \frac{1}{2} \left[ 1 + \tanh\left( \frac{\alpha - \alpha_y}{4 \varepsilon_y} \right) \right] \]  

(21)

depend on the velocity \( \alpha \) non-dimensionalised with respect to its amplitude, the yield parameter \( \alpha_y \) which is correlated with the fluid’s yield point and a smoothing parameter \( \varepsilon_y \) (Kamath and Wereley 1997b). A scheme of the force-displacement relationship is shown in Figure 14; \( L_{ve} \) and \( L_{vi} \) are the linear operators representing the Equations (17) and (18), respectively.

![Figure 14. Scheme of the viscoelastic-plastic model (Kamath and Wereley 1997a).](image)

The values for the parametric constants were found to be strong functions of the electric field, and it was proposed to approximate the coefficients associated with the viscoelastic and plastic properties as polynomial functions
of the field strength. Comparisons between predicted and experimental data showed that the model is able to reproduce the non-linear effects of the ER behaviour qualitatively. In addition, the model is numerically robust due to the linearity of the parallel shear flow mechanisms (Kamath and Wereley 1997a).

3.1.8 Augmented Non-Linear Viscoelastic-Plastic Model
(Kamath and Wereley 1997b)

To further reproduce the force-velocity characteristic of the considered ER fluid device Kamath and Wereley (1997b) extended the non-linear model described above. In the pre-yield region the friction force $F_c$ weighted by a shape function $S_c$ was added to allow for Coulomb-like sticktion effects observed at low velocities. Thus, the force $F_{by}$ generated in the pre-yield branch is given by

$$F_{by} = F_{vc} + S_c F_c$$

(22)

where

$$S_c = \frac{1}{2} \tanh \left( \frac{\dot{X}}{4 \varepsilon_c} \right),$$

(23)

and $\varepsilon_c$ is a smoothing factor. The viscoelastic component $F_{vc}$ is governed by Equation (17).

![Inertial mechanism of the augmented viscoelastic-plastic model](image)

Figure 15. Inertial mechanism of the augmented viscoelastic-plastic model (Kamath and Wereley 1997b).

To account for fluid inertia effects beyond the yield point Kamath and Wereley introduced the viscous and inertial mechanism depicted in Figure 15. Thus, the force $F_{ag}$ in the post-yield branch is given by

$$F_{ag} = C_v \dot{X} + R \ddot{X}.$$  

(24)
Both shear flow mechanisms are combined by the two non-linear weighting functions $S_{by} = S_{ce}$ (Eq. 20) and $S_{ay} = S_{ei}$ (Eq. 21) yielding the non-linear network depicted in Figure 16. The total force $F$ generated by this augmented viscoelastic-plastic model is given by

$$F = S_{by}F_{by} + S_{ay}F_{ay}.$$  

(25)

A comparison between the force-velocity characteristic predicted by the proposed model and obtained from experimental results is provided in Figure 17. The model precisely depicts the behaviour of the considered rheological device at different field strengths and displacement amplitudes (Kamath and Wereley 1997b). The added mechanisms, such as the friction component $F_c$, largely depend on the design of the considered damper, but they can be adjusted by choosing suitable parameter values. Moreover, the non-linear combination of linear flow mechanisms is numerically tractable.

Figure 17. Comparison between the predicted (——) and the experimentally obtained (••••) force-velocity characteristic for the augmented viscoelastic-plastic model (Kamath and Wereley 1997b).
3.1.9 Other Models

For the analysis of rheological fluid devices operating in the squeeze-flow mode, the equation governing the shear stress $\tau$ in the Bingham plastic model can be generalised to the bi-viscous relationship

$$
\tau = \begin{cases} 
\eta_p \dot{\gamma}, & |\tau| < \tau_1 \\
\tau_0 + \eta \dot{\gamma}, & |\tau| > \tau_1
\end{cases}
$$

(26)

where $\dot{\gamma}$ is the strain rate, and $\eta_p$ and $\eta$ are related to the elastic and the viscous fluid properties respectively (Stanway et al. 1996). The yield parameters $\tau_0$ and $\tau_1$ satisfy

$$\tau_0 = \tau_1 \left(1 - \frac{\eta_p}{\eta} \right),$$

(27)

as it is illustrated in Figure 18. The Bingham plastic model is obtained for $\eta_p \to \infty$.

![Figure 18. Bi-viscous model (Stanway et al. 1996).](image)

Makris et al. (1996a) developed a continuum mechanics constitutive model to characterise the behaviour of an ER prototype damper. The fluid's motion in the valve of the damper was approximated by Hagen-Poiseuille flow theory assuming laminar, one-dimensional flow through a stationary annular duct. The authors derived a linear first-order equation with variable coefficients to account for the elastic-viscoplastic properties of the fluid:

$$
\frac{d\tau}{dt} + \frac{G\dot{\gamma}}{\eta_0 \dot{\gamma} + \tau_p \text{sgn}(\dot{\gamma})} \cdot \tau = G\dot{\gamma}
$$

(28)

where $\eta_0$, $\tau_p$, and $G$ denote the plastic viscosity, the yield stress, and the elastic shear modulus of the fluid respectively. However, the model considerably depends on the physical properties of the fluid and the design of the damper (Makris et al. 1996b).
3.2 Non-Parametric Models

Non-parametric models are entirely based on the performance of a specific rheological fluid device. Commonly an elevated amount of experimental data, obtained by observing the rheological response to different excitations under varying operating conditions, is used to predict the device’s response to random excitations.

3.2.1 Chebyshev Polynomial Fit

Ehrgott and Masri (1992) used a Chebyshev polynomial fit to approximate the force generated by an ER test device. For fixed electric field strength (and fixed exciting frequency) the restoring force $F$ of the rheological device was predicted by an analytical function $\hat{F}$ constructed by two-dimensional orthogonal Chebyshev polynomials

$$F(x, \dot{x}) \approx \hat{F}(x, \dot{x}) = \sum_{i,j=0}^{m} C_{ij} T_i(x') T_j(\dot{x}')$$

where the $C_{ij} \in \mathbb{R}$ denote the two-dimensional Chebyshev coefficients, and $m$ is the degree of the polynomial. The values $x'$ and $\dot{x}'$ are obtained by normalising the displacement $x$ and the velocity $\dot{x}$ that are associated with the external excitation to the interval $[-1, +1]$. In the same way, the force $F$ can be determined as a function of the velocity $\dot{x}$ and the acceleration $\ddot{x}$.

Gavin et al. (1996b) extended this curve-fitting method to three dimensions. They related the restoring force $F$ of an ER fluid damper to the displacement $x$, the velocity $\dot{x}$ and the electric field strength $E$:

$$F(x, \dot{x}, E) \approx \hat{F}(x, \dot{x}, E) = \sum_{i,j,k=0}^{m} C_{ijk} T_i(x') T_j(\dot{x}') T_k(E')$$

where the $C_{ijk} \in \mathbb{R}$ denote the Chebyshev coefficients, and $x'$, $\dot{x}'$ and $E'$ are normalised values. For the purpose of controlling rheological fluid devices the electric field strength may also be approximated by a function $\hat{E}(x, \dot{x}, |F|)$ of the displacement, the velocity and the (desired) damping force.

A comparison between a force-velocity characteristic approximated by Chebyshev polynomials and the result of experiments conducted by Ehrgott and Masri (1992) is provided in Figure 19. The predicted ER response resembles the corresponding experimental data. However, the force plots published by Ehrgott and Masri partly exhibit oscillatory behaviour frequently
observed for polynomial interpolation. In addition, Kamath and Wereley (1997a) pointed out the computational effort to determine the large number of Chebyshev coefficients.

![Force vs Velocity](image)

Figure 19. Comparison between the predicted (……..) and the experimentally obtained (———) force-velocity characteristic for a Chebyshev polynomial fit (Ehrgott and Masri 1992).

### 3.2.2 Neural Networks

Burton et al. (1996) analysed the performance of a multilayer neural network to predict the electromechanical response. Neural networks consist of several processing units (neurons) whose inputs are weighted and passed to an activation (signal) function producing one single output. The weighting depends on the strength of the neurons' interconnections and can be adjusted by a kind of learning process (Burton et al. 1996). The network presented by Burton et al. was constructed by an algorithm known as the Dependence Identification Algorithm which is attributed to Moody and Antsaklis\(^1\). It was trained with different earthquake displacement histories and the corresponding responses of the considered seismic ER damper at varying field strengths.

Makris et al. (1996b) extended the use of neural networks to a combination with mechanical models mentioned earlier. As the latter were assumed to reproduce most of the linear ER response, the above network was trained on the difference signal between the response predicted by the parametric models and the actual response of the damper.

Burton et al. (1996) found that the performance of the mere neural network was surpassed by discrete element models such as the Bingham model. When combined with simple mechanical models the network's prediction was

partly superior to the results achieved with parametric methods alone. However, a conjunction with the more sophisticated BingMax model showed no improvements in simulating the ER fluid device (Makris et al. 1996b). A comparison between experimental data and a prediction obtained from the neural network combined with the parametric Maxwell model is shown in Figure 20.

![Graph showing force versus velocity](image)

Figure 20. Comparison between the predicted (——) and the experimentally obtained (....... ) force-velocity characteristic for a neural network combined with the Maxwell model (Makris et al. 1996b).

4 Numerical Simulation of a Passive Rheological Vibration Damper

In this section, simulation results are presented for a vehicle suspension design containing a rheological vibration damper at a constant field strength, i.e., a passive rheological damper; the latter was represented by mechanical models described above. The simulations conducted for a quarter vehicle model (cf. Figure 21) subject to different road excitations have been performed with MATLAB/SIMULINK 2.0².

4.1 Quarter Vehicle Model

![Quarter vehicle model with a passive (rheological) damper.](image)

The equations of motion for the above depicted quarter vehicle model can be derived as

\[
m_1 \ddot{u}_1 = K \cdot (u_0 - u_1) + C \cdot (\dot{u}_0 - \dot{u}_1) + k \cdot (u_2 - u_1) + F_{rh} \tag{31}
\]

\[
m_2 \ddot{u}_2 = -k \cdot (u_2 - u_1) - F_{rh}, \tag{32}
\]

where \(m_1\) and \(m_2\) are the masses of the axle and the vehicle body, \(u_1\) and \(u_2\) denote their vertical displacement, and \(u_0\) is the road disturbance. Furthermore, \(K\) and \(C\) represent the spring and damping constants of wheel and tyre, \(k\) denotes the stiffness of the suspension, and \(F_{rh}\) is the force exerted by the rheological damper.

The mass parameters \(m_1 = 250\) kg and \(m_2 = 1300\) kg used for the following simulations correspond to the values of a small bus or utility vehicle; the stiffness \(K = 650000\) N/m and the damping rate \(C = 800\) Ns/m of the tyre, as well as the spring constant \(k = 110000\) N/m of the suspension have been adapted as for a typical ground vehicle (cf. Hač 1992, Margolis and Goshtasbpour 1984).

The rheological damper is reproduced by the Bingham model and the modified Bouc-Wen model described in Sections 3.1.1 and 3.1.6. The performance of the quarter vehicle model subject to two different road disturbances is investigated (cf. Koslik et al. 1998).

A simple, conventional damper model is used for comparison. Its exerted force is given by

\[
F = c \cdot (\dot{u}_2 - \dot{u}_1) \tag{33}
\]
where the damping constant was chosen as $c = 6000 \text{ Ns/m}$.

### 4.2 Bingham Model of a Passive Rheological Damper

In case of the Bingham model, the force generated by the rheological damper results from Equation (2) as

$$F_r = f_c \cdot \text{sgn}(\dot{u}_2 - \dot{u}_1) + c_0 (\dot{u}_2 - \dot{u}_1).$$  \hspace{1cm} (34)

The parameter values used in the simulations correspond to the experimental data presented by Spencer et al. (1997) for the prototype of a magnetorheological fluid damper. Thus, the parameters related to the fluid’s viscosity and the yield stress have been chosen as $c_0 = 5000 \text{ Ns/m}$ and $f_c = 670 \text{ N}$.

### 4.3 Bouc-Wen Model of a Passive Rheological Damper

The equations governing the force of the rheological damper in case of the modified Bouc-Wen model can be derived from the Equations (10), (11) and (12) yielding

$$F_r = c_1 (\dot{y} - \dot{u}_1) + k_1 [ (u_2 - u_1) - x_0 ]$$  \hspace{1cm} (35)

where

$$\dot{y} = \frac{1}{c_0 + c_1} [ \alpha z + c_0 \ddot{u}_2 + c_1 \dot{u}_1 + k_0 (u_2 - y) ]$$  \hspace{1cm} (36)

and

$$\dot{z} = -\gamma |\dot{u}_2 - \dot{y}| z |z|^{n-1} - \beta (\dot{u}_2 - \dot{y}) |z|^n + \delta (\dot{u}_2 - \dot{y}).$$  \hspace{1cm} (37)

Likewise the parameters in this model have been chosen according to the data presented by Spencer et al. (1997) thus, $c_0 = 5300 \text{ Ns/m}$, $c_1 = 93000 \text{ Ns/m}$, $k_0 = 1400 \text{ N/m}$, $k_1 = 540 \text{ N/m}$, $\alpha = 96300 \text{ N/m}$, $\beta = 2000000 \text{ m}^{-2}$, $\gamma = 2000000 \text{ m}^{-2}$, $n = 2$, $\delta = 207$ and $x_0 = 0$. 
4.4 Simulation Results for a Step Disturbance

Numerical results for the first disturbance resulting from a step with a height of 0.1 m at time $t = 0$ (cf. Koslik et al. 1998) are depicted in Figs. 22–24.

For the passive rheological damper a faster decay of the excitation, especially in $u_2$ is achieved as compared to the conventional device (Fig. 22). No significant difference is observed between the behavior of the Bingham and the modified Bouc-Wen model.

![Graph](image)

**Figure 22.** Simulation results for the quarter vehicle model subject to a step of height 0.1 m at initial time. Here, the performance of the rheological damper represented by the Bingham model with respect to $u_2$ and $\dot{u}_2$ is quite similar to the modified Bouc-Wen model (first row). A conventional viscous damper is used for comparison (second row). Left column: Vertical displacement of the vehicle body. Right column: Vertical velocity of the vehicle body.
Figure 23. Simulation results for the quarter vehicle model subject to a step of height 0.1m at initial time. The rheological damper is represented by the Bingham model (first row) and the modified Bouc-Wen model (second row). A conventional viscous damper is used for comparison (third row). Left column: Relative velocity between the vehicle body and the axle. Right column: Zoom into the force-velocity characteristic.
Figure 24. Simulation results for the quarter vehicle model subject to a step of height 0.1 m at initial time. The rheological damper is represented by the modified Bouc-Wen model (first row). The behavior of the Bingham model is quite similar and thus omitted here. A conventional viscous damper is used for comparison (second row). Left column: Vertical displacement of the vehicle axle. Right column: Vertical velocity of the vehicle axle.

4.5 Simulation Results for a Sinusoidal Bump

The second disturbance consists of a sinusoidal bump of the same maximum height of 0.1 m as depicted in Figure 25. The corresponding numerical results are displayed in Figs. 26–28.

The damping of the vibration is again faster for the passive rheological damper than for the conventional damper. For the sinusoidal bump, a slightly different behavior of the Bingham and the modified Bouc-Wen model can be observed, especially for the state $u_2$ (Fig. 28).
Figure 25. Disturbance signal simulating the ride over a sinusoidal bump with a maximum height of 0.1 m.

Figure 26. Simulation results for the quarter vehicle model subject to a sinusoidal bump of height 0.1 m. The rheological damper is represented by the modified Bouc-Wen model (first row). The behavior of the Bingham model is quite similar and thus omitted here. A conventional viscous damper is used for comparison (second row). Left column: Vertical displacement of the vehicle axle. Right column: Vertical velocity of the vehicle axle.
Figure 27. Simulation results for the quarter vehicle model subject to a sinusoidal bump of height 0.1m. The rheological damper is represented by the Bingham model (first row), the modified Bouc-Wen model (second row). A conventional viscous damper is used for comparison (third row). Left column: Relative velocity between vehicle body and axle. Right column: Zoom into force-velocity characteristic.
Figure 28. Simulation results for the quarter vehicle model subject to a sinusoidal bump of height 0.1m. The rheological damper is represented by the Bingham model (first row), the modified Bouc-Wen model (second row). A conventional viscous damper is used for comparison (third row). Left column: Vertical displacement of the vehicle body. Right column: Vertical velocity of the vehicle body.
5 Further Applications of Rheological Fluid Devices

Adaptively controllable ER fluid devices to be used as shock absorbers in vehicles are described by Hartsock et al. (1991) and Petek (1992). Other automotive and related applications such as rheological clutches or engine mounts are investigated by Hartsock et al. (1991), Lampe et al. (1998) and Whittle et al. (1995). Backé et al. (1997) developed an ER fluid servo drive in a joint project with Bayer AG and Carl Schenck AG. Adaptively controllable MR fluid devices are now available for semi-active vibration damping of driver seats in trucks and as rotational brakes for controllable resistance in aerobic exercise equipment (Lord Corporation 1997, Jolly et al. 1998). An ER fluid damper for semi-active control of vibrations of a flexible, lightweight robot arm is described by Li et al. (1995). MR fluid devices for seismic protection of structures are investigated by Carlson and Spencer (1996), Dyke et al. (1997) and Lord Corporation (1997). ER fluid devices for this purpose are described by Burton et al. (1996). Choi et al. (1990) investigate the vibration characteristics of a composite beam filled with an electrorheological fluid. The application of electroviscous fluids as movement sensor control devices in active vibration dampers is discussed by Oppermann et al. (1990). MR fluid lag mode dampers for helicopters for hingeless and bearingless rotors for improvint aeromechanical stability with respect to air and ground resonance are mentioned by Kamath, Wereley and Jolly (1997).

Many more designs and applications of electro- and magnetorheological fluid devices can be expected in the near future.

6 Conclusions and Outlook

The basic properties of electro- and magnetorheological fluids as well as various models for rheological fluid devices and their applications have been discussed in this paper. However, there is still a need for rigouros mathematical models that accurately describe rheological fluids and devices and are suitable for understanding, investigating and predicting their behaviour by numerical simulation.

Furthermore, to fully utilize the innovative potential of adaptively controllable rheological fluid devices, (optimal) control strategies must be developed taking into account the dynamical behaviour of the specific complex
mechanical system including the controllable rheological fluid device. For example, when using an active suspension based on controllable rheological fluid dampers, the full dynamical behavior of the vehicle, the damper and the road disturbances must be considered in order to maximize the driving comfort and safety of a car by adaptive damping and level control. For this purpose, models for actively controllable fluid devices must be developed describing the dynamical behaviour of a specific device with respect to changes in the electric or magnetic field.

Acknowledgement. The authors gratefully acknowledge the support by the German science foundation Deutsche Forschungsgemeinschaft (DFG) within the collaborative research center Sonderforschungsbereich 438. The authors also gratefully acknowledge the support given by Prof. Dr. Dr.h.c. R. Bulirsch (Technische Universität München). The authors are indebted for helpful and stimulating discussions to Prof. Dr. B.F. Spencer, Jr. (University of Notre Dame, Illinois), Dr. H. Rosenfeldt (Schenck Pegasus GmbH, Darmstadt), Dr. H. Böse (Fraunhofer Institut für Silicatforschung, Würzburg), Prof. Dr. R.H.W. Hoppe (Universität Augsburg), and Dipl.-Math. U. Rettig (Technische Universität München).

References


H. Böse, Private communication, (October 1998).


http://www-lit.mathematik.tu-muenchen.de/veroeff/html/SFB/982.49003.html

D. Lampe, *Materials Database on Commercially Available Electro- and Magnetorheological Fluids*, Institut für Luft- und Raumfahrttechnik, Technische Universität Dresden (1997), World Wide Web:
http://www.tu-dresden.de/mw/ir/lampe/HAUENG.HTM


N. N., *Provisional product information Rheobay TP AI 3565 and Rheobay TP AI 3566*, Bayer AG, Leverkusen, Germany (1994).


M. Whittle, R. J. Atkin and W. A. Bullough, *Fluid dynamic limitations